

Workshop 10.5a: Logistic regression

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Section 1

Logistic
regression

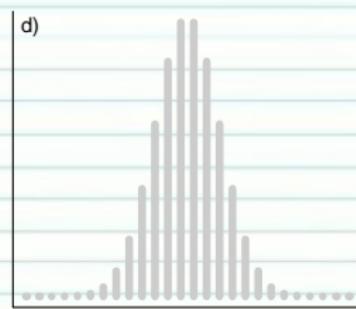
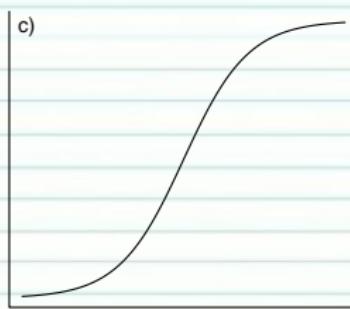
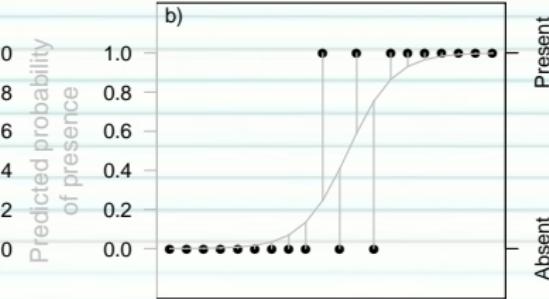
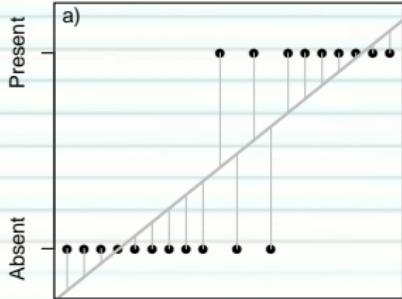
Logistic regression

BINARY DATA

Link: $\log \left(\frac{\pi}{1-\pi} \right)$

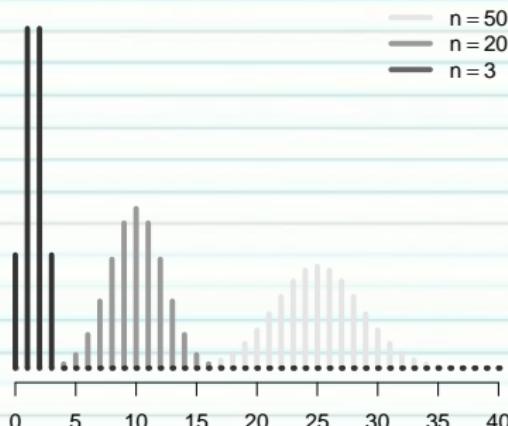
Transform scale of linear predictor $(-\infty, \infty)$ into that of the response $(0,1)$

Logistic regression

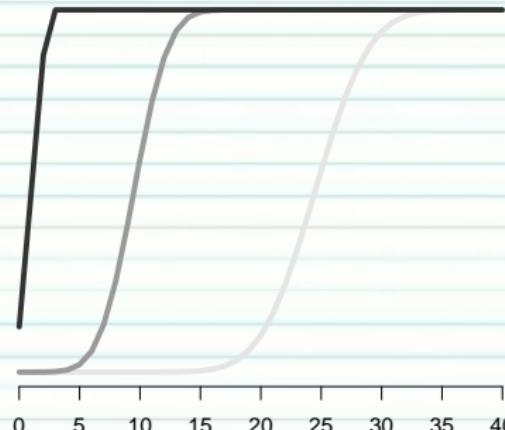


Logistic regression

Probability density function



Cumulative density function



$$E(Y) = \binom{n}{x} p^x (1-p)^{n-x}$$

Spread assumed to be equal to mean. ($\phi = 1$)

Dispersion

OVER-DISPERSION

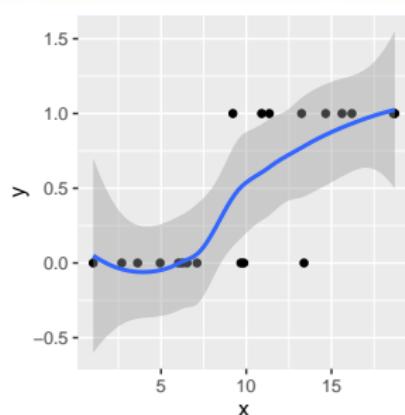
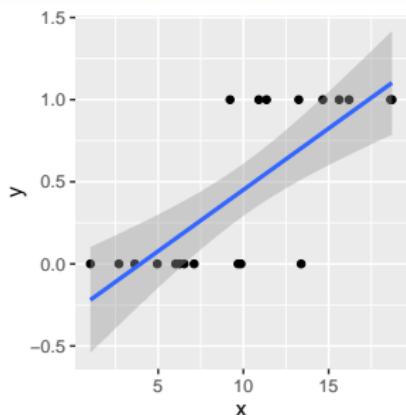
Sample more varied than expected from its mean

- variability due to other unmeasured influences
 - quasi-model
- due to more zeros than expected
 - zero-inflated model

Logistic regression

Example data

y	x
1	0 1.024733
2	0 2.696719
3	0 3.626263
4	0 4.948643
5	0 6.024718
6	0 6.254113



Logistic regression

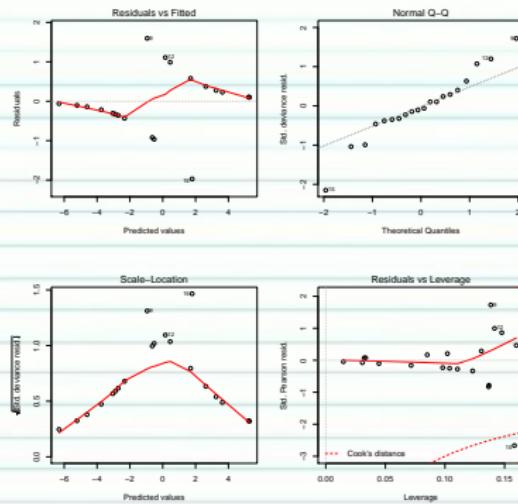
- Fit model

```
> dat.glmL <- glm(y ~ x, data = dat, family = "binomial")
```

Logistic regression

- Explore residuals

```
> par(mfrow=c(2,2))  
> plot(dat.glmL)
```



Logistic regression

- Explore goodness of fit
- Pearson χ^2 residuals

```
> dat.resid <- sum(resid(dat.glmL, type = "pearson")^2)
> 1 - pchisq(dat.resid, dat.glmL$df.resid)
```

[1] 0.8571451

- Deviance (G^2)

```
> 1-pchisq(dat.glmL$deviance, dat.glmL$df.resid)
```

[1] 0.8647024

Logistic regression

- Explore model parameters

Slope parameter is on log odds-ratio scale

```
> summary(dat.glmL)
```

Call:

```
glm(formula = y ~ x, family = "binomial", data = dat)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-1.97157	-0.33665	-0.08191	0.30035	1.59628

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-6.9899	3.1599	-2.212	0.0270 *
x	0.6559	0.2936	2.234	0.0255 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 27.526 on 19 degrees of freedom

Logistic regression

- Quasi R²

$$\text{quasiR}^2 = 1 - \left(\frac{\text{deviance}}{\text{null deviance}} \right)$$

```
> 1-(dat.glmL$deviance/dat.glmL>null)
```

```
[1] 0.5767057
```

Logistic regression

- LD50

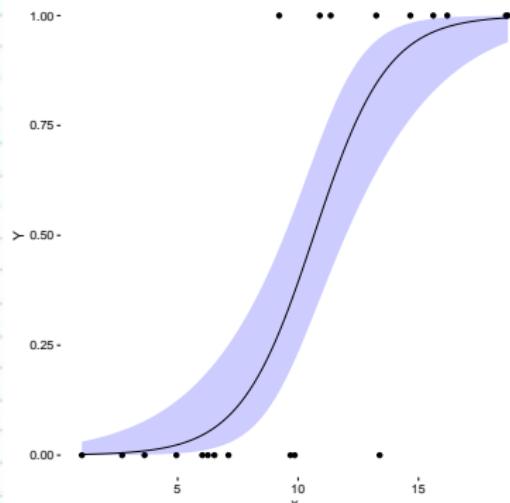
$$LD50 = -\frac{\text{intercept}}{\text{slope}}$$

```
> -dat.glmL$coef[1]/dat.glmL$coef[2]
```

(Intercept)
10.65781

Logistic regression

- summary figure



Section 2

Worked
Examples

Worked Examples