Workshop 11.2a: Generalized Linear Mixed Effects Models (GLMM)

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1. Generalized Linear Mixed Effects Models

1.1. Parameter Estimation

lm –> LME
(integrate likelihood across all unobserved levels random effects)

1.2. Parameter Estimation

lm –> LME
(integrate likelihood across all unobserved levels random effects)

glm —-. . . . . –> GLMM
Not so easy - need to approximate

1.3. Parameter Estimation

- Penalized quasi-likelihood
- Laplace approximation
- Gauss-Hermite quadrature

1.4. Penalized quasi-likelihood (PQL)

1.4.1. Iterative (re)weighting

- LMM to estimate vcov structure
- fixed effects estimated by fitting GLM (incorp vcov)
- refit LMM to re-estimate vcov
- cycle

1.5. Penalized quasi-likelihood (PQL)

1.5.1. Advantages

- relatively simple
- leverage variance-covariance structures for heterogeneity and dependency structures

1.5.2. Disadvantages

- biased when expected values less <5
- approximates likelihood (no AIC or LTR)

1.6. Laplace approximation

Second-order Taylor series expansion - to approximate likelihood at unobserved levels of random effects

1.7. Laplace approximation

Second-order Taylor series expansion - to approximate likelihood at unobserved levels of random effects

1.7.1. Advantages

- more accurate

1.8. Laplace approximation

Second-order Taylor series expansion - to approximate likelihood at unobserved levels of random effects

1.8.1. Advantages

- more accurate

1.8.2. Disadvantages

- slower
- no way to incorporate vcov

1.9. Gauss-Hermite quadrature (GHQ)

- approximates value of integrals at specific points (quadratures)
- points (and weights) selected by optimizer

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1.10.1. Advantages

- even more accurate

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- approximates value of integrals at specific points (quadratures)
- points (and weights) selected by optimizer

1.11.1. Advantages

- even more accurate

1.11.2. Disadvantages

- even slower
- no way to incorporate vcov

1.12. Markov Chain Monte Carlo (MCMC)

- recreate likelihood by sampling proportionally to likelihood

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1.13.1. Advantages

- very accurate (not an approximation)
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1.14. Markov Chain Monte Carlo (MCMC)

- recreate likelihood by sampling proportionally to likelihood

1.14.1. Advantages

- very accurate (not an approximation)
- very robust

1.14.2. Disadvantages

- very slow
- currently complex

1.15. Inference (hypothesis) testing

1.15.1. GLMM

Depends on:

- Estimation engine (PQL, Laplace, GHQ)
- Overdispersed
- Fixed or random factors

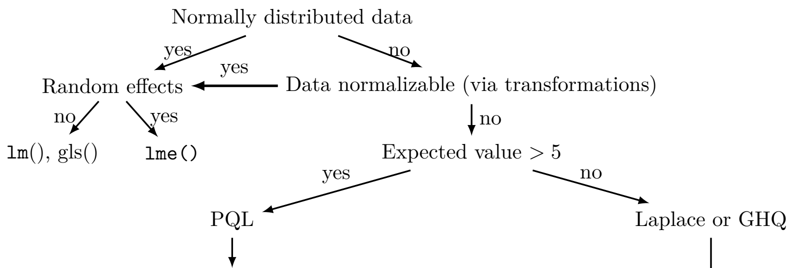
1.16. Inference (hypothesis) testing

Approximation	Characteristics	Associated inference	R Function
Penalized Quasi-likelihood (PQL)	Fast and simple, accommodates heterogeneity and dependency structures, biased for small samples	Wald tests only	`glmmPQL` (MASS)
Laplace	More accurate (less biased), slower, does not accommodate heterogeneity and dependency structures	LRT	`glmer` (lme4), `glmmadmb` (glmmADMB)
Gauss-Hermite quadrature	Evan more accurate (less biased), slower, does not accommodate heterogeneity and dependency structures, cant handle more than 1 random effect	LRT	`glmer` (lme4)?? - does not seem to work
Markov Chain Monte Carlo (MCMC)	Bayesian, very flexible and accurate, yet very slow and more complex	Bayesian credibility intervals, Bayes factors	Numerous (see Tutorial 9.2b)

Feature	`glmmQPL` (MASS)	`glmer` (lme4)	`glmmadmb` (glmmADMB)	MCMC
Varoamce amd covariance structures	Yes	-	not yet	Yes
Overdispersed (Quasi) families	Yes	limited	some	-
Mixture families	limited	limited	limited	Yes
Zero-inflation	-	-	Yes	Yes
Residual degrees of freedom	Between-within	-*	-	NA
Parameter tests	Wald t	Wald Z	Wald Z	UI
Marginal tests (fixed effects)	Wald F, χ^2	Wald F, χ^2	Wald F, χ^2	UI
Marginal tests (random effects)	Wald F, χ^2	LRT	LRT	UI
Information criterion	-	AIC	AIC	AIC, WAIC

1.17. Inference (hypothesis) testing

1.18. Inference (hypothesis) testing



Overdispersed	Model	Inference
No	`glmmPQL()`	Wald Z or χ^2
Yes	`glmmPQL(.., family='quasi..')`	Wald t or F
Clumpiness	`glmmPQL(.., family='negative.binomial')`	Wald t or F
Zero-inflation	`glmmadmb(.., zeroInflated=TRUE)`	Wald t or F

Overdispersed	Model	Inference
Random effects		
Yes or no	`glmer()` or `glmmadmb()`	LRT (ML)
Fixed effects		
No	`glmer()` or `glmmadmb()`	Wald Z or χ^2
Yes	`glmer(..(1|Obs))`	Wald t or F
Clumpiness	`glmer(.., family='negative.binomial')`	Wald t or F
	`glmmamd(.., family='nbinom')`	Wald t or F
Zero-inflation	`glmmadmb(.., zeroInflated=TRUE)`	Wald t or F

1.19. Additional assumptions

- dispersion
- (multi)collinearity
- design balance and Type III (marginal) SS
- heteroscadacity
- spatial/temporal autocorrelation

2. Worked Examples

2.1. Worked Examples

$$log(y_{ij}) = \gamma_{Site_i} + \beta_0 + \beta_1 Treat_i + \varepsilon_{ij} \qquad \varepsilon \sim Pois(\lambda)$$

where $\sum \gamma = 0$