Workshop 3.2: Matrices

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Matrices

y_1	y_2	y_3	y_4	$\cdots$	y_j	$\cdots$	y_p
y_{11}	y_{12}	y_{13}	y_{14}	$\cdots$	y_{1j}	$\cdots$	y_{1p}
y_{21}	y_{22}	y_{23}	y_{24}	$\cdots$	y_{2j}	$\cdots$	y_{2p}
y_{31}	y_{32}	y_{33}	y_{34}	$\cdots$	y_{3j}	$\cdots$	y_{3p}
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdots$	$\cdot$	$\cdots$	$\cdot$
y_{i1}	y_{i2}	y_{i3}	y_{i4}	$\cdots$	y_{ij}	$\cdots$	y_{ip}
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdots$	$\cdot$	$\cdots$	$\cdot$
y_{n1}	y_{n2}	y_{n3}	y_{n4}	$\cdots$	y_{nj}	$\cdots$	y_{np}

$$Y = [y_{ij}] = \begin{bmatrix} y_{11} & y_{12} & \cdot & \cdot & \cdot & y_{1p} \\ y_{21} & y_{22} & \cdot & \cdot & \cdot & y_{2p} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

Constructing matrices

```
> matrix(c(1,5,10,0,
+          4,2,3,6,
+          1,7,4,1),nrow=4,ncol=3)
```

```
     [,1] [,2] [,3]
[1,]    1    4    1
[2,]    5    2    7
[3,]   10    3    4
[4,]    0    6    1
```

```
> M<-matrix(c(1,5,10,
+ 0,4,2,
+ 3,6,1,
+ 7,4,1),nrow=4,ncol=3,byrow=TRUE)
> M
```

```
     [,1] [,2] [,3]
[1,]    1    5   10
[2,]    0    4    2
[3,]    3    6    1
[4,]    7    4    1
```

Properties

```
> class(M)
```

```
[1] "matrix"
```

Properties

ORDER = DIMENSIONS

```
> class(M)
```

```
[1] "matrix"
```

```
> #dimensions
> dim(M)
```

```
[1] 4 3
```

```
> nrow(M)
```

```
[1] 4
```

```
> ncol(M)
```

Properties

```
> class(M)
```

```
[1] "matrix"
```

```
> #dimensions
> dim(M)
```

```
[1] 4 3
```

```
> nrow(M)
```

```
[1] 4
```

```
> ncol(M)
```

```
[1] 3
```

```
> colnames(M) <- c('Sp1','Sp2','Sp3')
> rownames(M) <- paste('Site',1:4)
> M
```

```
       Sp1 Sp2 Sp3
```

Transposing matrices

If a matrix (Y) is:

$$Y = [y_{ij}] = \begin{bmatrix} 7 & 18 & -2 & 6 \\ 3 & 55 & 1 & 9 \\ -4 & 0 & 31 & 7 \end{bmatrix}$$

Then the transposed matrix (Y[?]) is:

$$Y = [y_{ji}] = \begin{bmatrix} 7 & 3 & -4 \\ 18 & 55 & 0 \\ -2 & 1 & 31 \\ 6 & 9 & 7 \end{bmatrix}$$

```
> Y <- matrix(c(7,18,-2,22
+ -16,3,55,1,
+ 9,-4,0,31
+ ),nrow=3,ncol=4,byrow=TRUE)
> Y
```

```
     [,1] [,2] [,3] [,4]
[1,]    7   18   -2    6
[2,]    3   55    1    9
[3,]   -4    0   31    7
```

Matrix algebra

Conformable matrices - same order

MATRIX ADDITION

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \; B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1+7 & 2+8 & 3+9 \\ 4+10 & 5+11 & 6+12 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

```
> (A <- matrix(c(1,2,3,4,5,6),nrow=2,ncol=3,byrow=TRUE))
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
```

Matrix algebra

Conformable matrices - same order

MATRIX SUBTRACTION

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

$$\begin{aligned} B-A &= B+(-1)A \\ &= \begin{bmatrix} 7+(-1) & 8+(-2) & 9+(-3) \\ 10+(-4) & 11+(-5) & 12+(-6) \end{bmatrix} \\ &= \begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \end{bmatrix} \end{aligned}$$

```
> (A <- matrix(c(1,2,3,4,5,6),nrow=2,ncol=3,byrow=TRUE))
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
```

Matrix algebra

Conformable - nrow(A)=ncol(B)

MATRIX MULTIPLICATION

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 \\ -3 & 2 \\ 0 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1 \times 1) + (2 \times -3) + (3 \times 0) & (1 \times 5) + (2 \times 2) + (3 \times \\ (4 \times 1) + (5 \times -3) + (6 \times 0) & (4 \times 5) + (5 \times 2) + (6 \times \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 \\ -11 & 24 \end{bmatrix}$$

```
> A <- matrix(c(1,2,3,4,5,6),nrow=2,ncol=3,byrow=TRUE)
> B <- matrix(c(1,-3,0,5,2,-1),nrow=3,ncol=2)
> A %*% B
```

Matrix algebra

Conformable - dim(A)=dim(B)

ELEMENT WISE PRODUCTS

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

$$A \odot B = \begin{bmatrix} 1 \times 7 & 2 \times 8 & 3 \times 9 \\ 4 \times 10 & 5 \times 11 & 6 \times 12 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 16 & 27 \\ 40 & 55 & 72 \end{bmatrix}$$

```
> A <- matrix(c(1,2,3,4,5,6),nrow=2,ncol=3,byrow=TRUE)
> B <- matrix(c(7,8,9,10,11,12),nrow=2,ncol=3, byrow=TRUE)
> A * B
```

```
     [,1] [,2] [,3]
```

Matrix algebra

SWEEPING

$$\mathtt{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \mathtt{B} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\mathtt{A} \times \mathtt{B} = \begin{bmatrix} 1 \times 2 & 2 \times 2 & 3 \times 2 \\ 4 \times 3 & 5 \times 3 & 6 \times 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 12 & 15 & 18 \end{bmatrix}$$

```
> A <- matrix(c(1,2,3,4,5,6),nrow=2,ncol=3,byrow=TRUE)
> B <- matrix(c(2,3), nrow=2, ncol=1, byrow=TRUE)
> sweep(A,1,B,'*')
```

```
     [,1] [,2] [,3]
[1,]    2    4    6
[2,]   12   15   18
```

Matrix algebra

SWEEPING

$$\mathtt{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \mathtt{B} = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

$$\mathtt{A} + \mathtt{B} = \begin{bmatrix} 1+2 & 2+3 & 3+4 \\ 4+2 & 5+3 & 6+4 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 7 \\ 6 & 8 & 10 \end{bmatrix}$$

```
> A <- matrix(c(1,2,3,4,5,6),nrow=2,ncol=3,byrow=TRUE)
> B <- matrix(c(2,3,4), nrow=1, ncol=3, byrow=FALSE)
> sweep(A,2,B,'+')
```

```
     [,1] [,2] [,3]
[1,]    3    5    7
[2,]    6    8   10
```

Matrix modifications via matrix algebra

MULTIPLYING ROWS