Presentation 7.3a: Multiple linear regression

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Section 1

Theory

Multiple Linear Regression

ADDITIVE MODEL

$$\text{growth} = \text{intercept} + \text{temperature} + \text{nitrogen}$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_j x_{ij} + \epsilon_i$$

OR

$$y_i = \beta_0 + \sum_{j=1:n}^{N} \beta_j x_{ji} + \epsilon_i$$

Multiple Linear Regression

ADDITIVE MODEL

$$\text{growth} = \text{intercept} + \text{temperature} + \text{nitrogen}$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_j x_{ij} + \epsilon_i$$

- effect of one predictor holding the other(s) constant

Multiple Linear Regression

ADDITIVE MODEL

$$\text{growth} = \text{intercept} + \text{temperature} + \text{nitrogen}$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_j x_{ij} + \epsilon_i$$

Y	X1	X2
3	22.7	0.9
2.5	23.7	0.5
6	25.7	0.6
5.5	29.1	0.7
9	22	0.8
8.6	29	1.3
12	29.4	1

Multiple Linear Regression

ADDITIVE MODEL

$$
\begin{aligned}
3 &= \beta_0 + (\beta_1 \times 22.7) + (\beta_2 \times 0.9) + \varepsilon_1 \\
2.5 &= \beta_0 + (\beta_1 \times 23.7) + (\beta_2 \times 0.5) + \varepsilon_2 \\
6 &= \beta_0 + (\beta_1 \times 25.7) + (\beta_2 \times 0.6) + \varepsilon_3 \\
5.5 &= \beta_0 + (\beta_1 \times 29.1) + (\beta_2 \times 0.7) + \varepsilon_4 \\
9 &= \beta_0 + (\beta_1 \times 22) + (\beta_2 \times 0.8) + \varepsilon_5 \\
8.6 &= \beta_0 + (\beta_1 \times 29) + (\beta_2 \times 1.3) + \varepsilon_6 \\
12 &= \beta_0 + (\beta_1 \times 29.4) + (\beta_2 \times 1) + \varepsilon_7
\end{aligned}
$$

Multiple Linear Regression

MULTIPLICATIVE MODEL

$$\text{growth} = \text{intercept} + \text{temp} + \text{nitro} + \text{temp} \times \text{nitro}$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2} + \ldots + \epsilon_i$$

Multiple Linear Regression

MULTIPLICATIVE MODEL

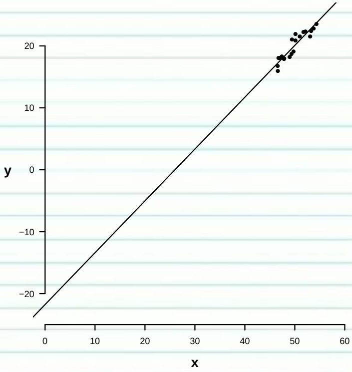
$$
\begin{aligned}
3 &= \beta_0 + (\beta_1 \times 22.7) + (\beta_2 \times 0.9) + (\beta_3 \times 22.7 \times 0.9) + \varepsilon_1 \\
2.5 &= \beta_0 + (\beta_1 \times 23.7) + (\beta_2 \times 0.5) + (\beta_3 \times 23.7 \times 0.5) + \varepsilon_2 \\
6 &= \beta_0 + (\beta_1 \times 25.7) + (\beta_2 \times 0.6) + (\beta_3 \times 25.7 \times 0.6) + \varepsilon_3 \\
5.5 &= \beta_0 + (\beta_1 \times 29.1) + (\beta_2 \times 0.7) + (\beta_3 \times 29.1 \times 0.7) + \varepsilon_4 \\
9 &= \beta_0 + (\beta_1 \times 22) + (\beta_2 \times 0.8) + (\beta_3 \times 22 \times 0.8) + \varepsilon_5 \\
8.6 &= \beta_0 + (\beta_1 \times 29) + (\beta_2 \times 1.3) + (\beta_3 \times 29 \times 1.3) + \varepsilon_6 \\
12 &= \beta_0 + (\beta_1 \times 29.4) + (\beta_2 \times 1) + (\beta_3 \times 29.4 \times 1) + \varepsilon_7
\end{aligned}
$$

Section 2

Centering data

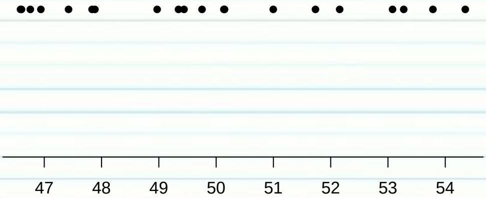
Multiple Linear Regression

CENTERING



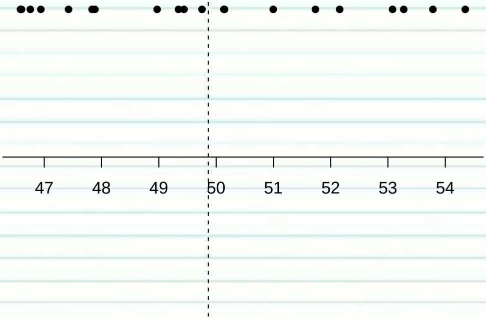
Multiple Linear Regression

CENTERING



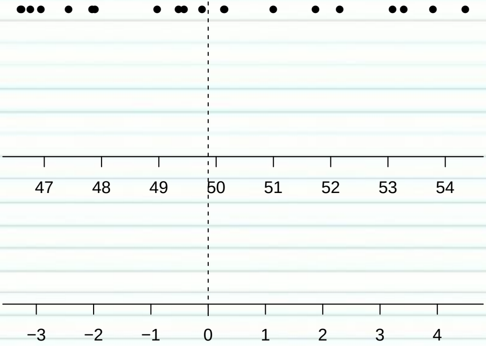
Multiple Linear Regression

CENTERING



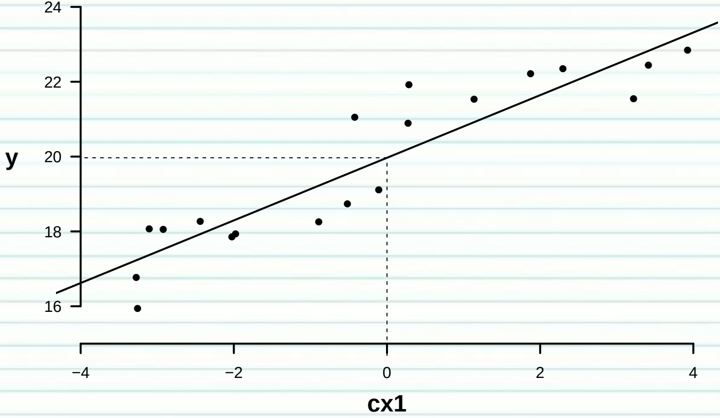
Multiple Linear Regression

CENTERING



Multiple Linear Regression

CENTERING



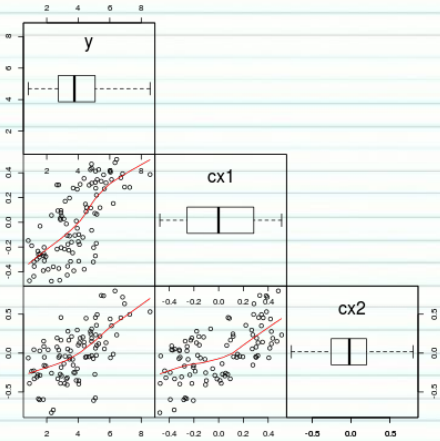
Section 3

Assumptions

Multiple Linear Regression

ASSUMPTIONS

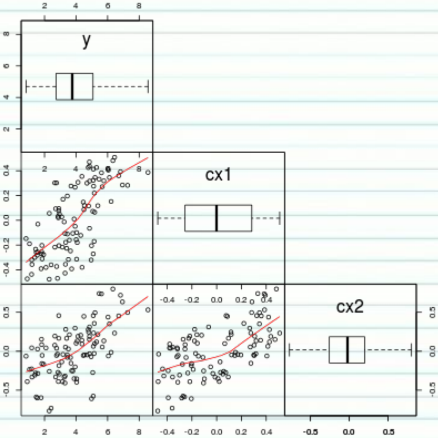
Normality, homog., linearity



Multiple Linear Regression

ASSUMPTIONS

(multi)collinearity



Multiple Linear Regression

VARIANCE INFLATION

Strength of a relationship

R^2

Strong when $R^2 \geq 0.8$

Multiple Linear Regression

VARIANCE INFLATION

$$\text{var.inf} = \frac{1}{1 - R^2}$$

Collinear when var.inf $>= 5$

Some prefer > 3

Multiple Linear Regression

ASSUMPTIONS

(multi)collinearity

```
library(car)
# additive model - scaled predictors
vif(lm(y ~ cx1 + cx2, data))
```

```
     cx1      cx2 
1.743817 1.743817 
```

Multiple Linear Regression

ASSUMPTIONS

(multi)collinearity

```
library(car)
# additive model - scaled predictors
vif(lm(y ~ cx1 + cx2, data))
```

```
     cx1      cx2 
1.743817 1.743817 
```

```
# multiplicative model - raw predictors
vif(lm(y ~ x1 * x2, data))
```

```
       x1        x2     x1:x2 
 7.259729  5.913254 16.949468 
```

Multiple Linear Regression

ASSUMPTIONS

```
# multiplicative model - raw predictors
vif(lm(y ~ x1 * x2, data))
```

```
       x1        x2     x1:x2 
 7.259729  5.913254 16.949468 
```

```
# multiplicative model - scaled predictors
vif(lm(y ~ cx1 * cx2, data))
```

```
     cx1      cx2  cx1:cx2 
1.769411 1.771994 1.018694 
```

Section 4

Multiple linear models in R

Model fitting

Additive model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$$

```
data.add.lm <- lm(y~cx1+cx2, data)
```

Model fitting

Additive model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$$

```
data.add.lm <- lm(y~cx1+cx2, data)
```

Multiplicative model

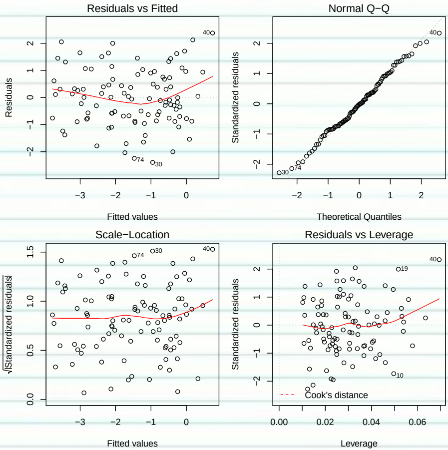
$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2} + \epsilon_i$$

```
data.mult.lm <- lm(y~cx1+cx2+cx1:cx2, data)
#OR
data.mult.lm <- lm(y~cx1*cx2, data)
```

Model evaluation

Additive model

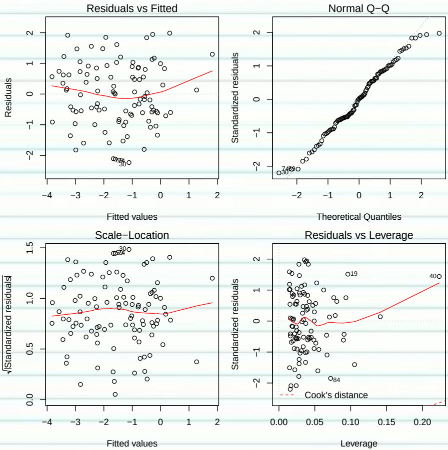
```
plot(data.add.lm)
```



Model evaluation

Multiplicative model

```
plot(data.mult.lm)
```



Model summary

Additive model

```
summary(data.add.lm)
```

```
Call:
lm(formula = y ~ cx1 + cx2, data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.39418 -0.75888 -0.02463  0.73688  2.37938 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -1.5161     0.1055 -14.364  < 2e-16 ***
cx1           2.5749     0.4683   5.499  3.1e-07 ***
cx2          -4.0475     0.3734 -10.839  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.055 on 97 degrees of freedom
Multiple R-squared:  0.5567,	Adjusted R-squared:  0.5476 
F-statistic: 60.91 on 2 and 97 DF,  p-value: < 2.2e-16
```

Model summary

Additive model

```
confint(data.add.lm)
```

```
                 2.5 %    97.5 %
(Intercept) -1.725529 -1.306576
cx1          1.645477  3.504300
cx2         -4.788628 -3.306308
```

Model summary

Multiplicative model

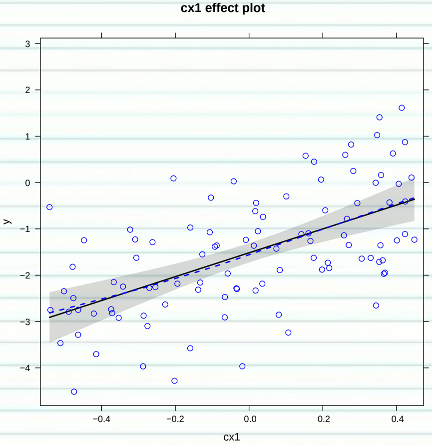
```r
summary(data.mult.lm)
```

```

Call:
lm(formula = y ~ cx1 * cx2, data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.23364 -0.62188  0.01763  0.80912  1.98568 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -1.6995     0.1228 -13.836  < 2e-16 ***
cx1           2.7232     0.4571   5.957 4.22e-08 ***
cx2          -4.1716     0.3648 -11.435  < 2e-16 ***
cx1:cx2       2.5283     0.9373   2.697  0.00826 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.023 on 96 degrees of freedom
Multiple R-squared:  0.588,	Adjusted R-squared:  0.5751 
F-statistic: 45.66 on 3 and 96 DF,  p-value: < 2.2e-16
```

Graphical summaries

Additive model

```
library(effects)
plot(effect("cx1",data.add.lm, partial.residuals=TRUE))
```



Graphical summaries

Additive model

```
library(effects)
library(ggplot2)
e <- effect("cx1",data.add.lm, xlevels=list(
      cx1=seq(-0.4,0.4, len=10)), partial.residuals=TRUE)
newdata <- data.frame(fit=e$fit, cx1=e$x, lower=e$lower,
      upper=e$upper)
resids <- data.frame(resid=e$partial.residuals.raw,
      cx1=e$data$cx1)
```

```
Error in data.frame(resid = e$partial.residuals.raw, cx1 = e$data$cx1): argumen
```

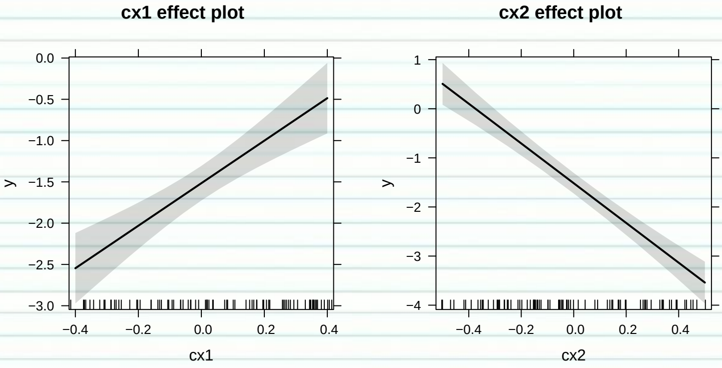
```
ggplot(newdata, aes(y=fit, x=cx1)) +
    geom_point(data=resids, aes(y=resid, x=cx1))+
    geom_ribbon(aes(ymin=lower, ymax=upper), fill='blue',
     alpha=0.2)+
    geom_line()+theme_classic()
```

```
Error in fortify(data): object 'resids' not found
```

Graphical summaries

Additive model

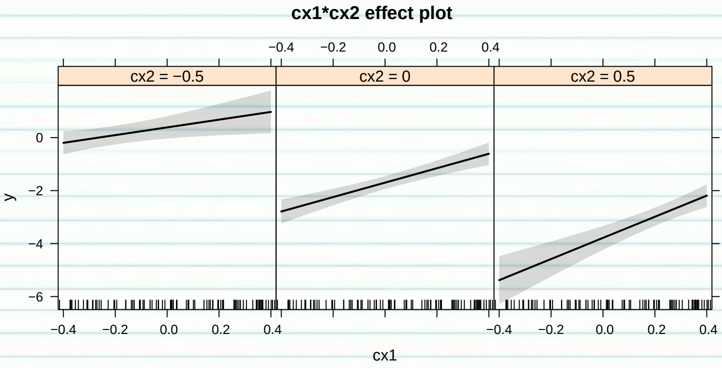
```
library(effects)
plot(allEffects(data.add.lm))
```



Graphical summaries

Multiplicative model

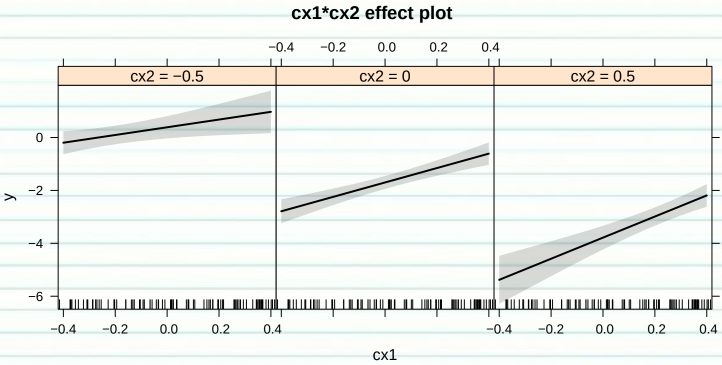
```
library(effects)
plot(allEffects(data.mult.lm))
```



Graphical summaries

Multiplicative model

```
library(effects)
plot(Effect(focal.predictors=c("cx1","cx2"),data.mult.lm))
```



Section 5

Model selection

How good is a model?

"All models are wrong, but some are useful" George E. P. Box

CRITERIA

- R^2 - no
- Information criteria
  - AIC, AICc
  - penalize for complexity

Model selection

CANDIDATES

```
AIC(data.add.lm, data.mult.lm)
```

```
             df      AIC
data.add.lm   4 299.5340
data.mult.lm  5 294.2283
```

```
library(MuMIn)
AICc(data.add.lm, data.mult.lm)
```

```
             df     AICc
data.add.lm   4 299.9551
data.mult.lm  5 294.8666
```

Model selection

DREDGING

```r
library(MuMIn)
data.mult.lm <- lm(y~cx1*cx2, data, na.action=na.fail)
dredge(data.mult.lm, rank="AICc", trace=TRUE)
```

```
0 : lm(formula = y ~ 1, data = data, na.action = na.fail)
1 : lm(formula = y ~ cx1 + 1, data = data, na.action = na.fail)
2 : lm(formula = y ~ cx2 + 1, data = data, na.action = na.fail)
3 : lm(formula = y ~ cx1 + cx2 + 1, data = data, na.action = na.fail)
7 : lm(formula = y ~ cx1 + cx2 + cx1:cx2 + 1, data = data, na.action = na.fail)


Global model call: lm(formula = y ~ cx1 * cx2, data = data, na.action = na.fail)
---
Model selection table
   (Int)     cx1    cx2 cx1:cx2 df   logLik  AICc delta weight
8 -1.699  2.7230 -4.172   2.528  5 -142.114 294.9  0.00  0.927
4 -1.516  2.5750 -4.047          4 -145.767 300.0  5.09  0.073
3 -1.516         -2.706          3 -159.333 324.9 30.05  0.000
1 -1.516                         2 -186.446 377.0 82.15  0.000
2 -1.516 -0.7399                 3 -185.441 377.1 82.27  0.000
Models ranked by AICc(x)
```

Multiple Linear Regression

MODEL AVERAGING

```
library(MuMIn)
data.dredge<-dredge(data.mult.lm, rank="AICc")
model.avg(data.dredge, subset=delta<20)
```

```
Call:
model.avg(object = data.dredge, subset = delta < 20)

Component models:
'123' '12'

Coefficients:
       (Intercept)      cx1       cx2   cx1:cx2
full     -1.686125 2.712397 -4.162525 2.344227
subset   -1.686125 2.712397 -4.162525 2.528328
```

Multiple Linear Regression

MODEL SELECTION

Or more preferable:

- identify 10-15 candidate models
- compare these via AIC (etc)

Section 6

Worked Examples

Worked examples

```r
loyn <- read.csv('../data/loyn.csv', strip.white=T)
head(loyn)
```

```
  ABUND AREA YR.ISOL DIST LDIST GRAZE ALT
1   5.3  0.1    1968   39    39     2 160
2   2.0  0.5    1920  234   234     5  60
3   1.5  0.5    1900  104   311     5 140
4  17.1  1.0    1966   66    66     3 160
5  13.8  1.0    1918  246   246     5 140
6  14.1  1.0    1965  234   285     3 130
```

Worked Examples

Question: what effects do fragmentation variables have on the abundance of forest birds

Linear model:

$$\mathrm{Abund}_i = \beta_0 + \sum_{j=1:n}^{N} \beta_j X_{ji} + \varepsilon_i \qquad \varepsilon \sim \mathcal{N}(0, \sigma^2)$$