Presentation 7.3b: Multiple linear regression

Murray Logan
August 9, 2016

Table of contents

0.1. Preparations

0.1.1. Packages

```
library(ggplot2)
library(car)
library(GGally)
library(rstan)
library(brms)
library(coda)
library(dplyr)
library(gridExtra)
```

0.1.2. Data

www.flutterbys.com.au/stats/downloads/data/loyn.csv

www.flutterbys.com.au/stats/downloads/data/paruelo.csv

1. Theory

1.1. Multiple Linear Regression

1.1.1. Additive model

$$growth = intercept + temperature + nitrogen$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_j x_{ij} + \epsilon_i$$

OR

$$y_i = \beta_0 + \sum_{j=1:n}^{N} \beta_j x_{ji} + \epsilon_i$$

1.2. Multiple Linear Regression

1.2.1. Additive model

$$growth = intercept + temperature + nitrogen$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_j x_{ij} + \epsilon_i$$

- effect of one predictor holding the other(s) constant

1.3. Multiple Linear Regression

1.3.1. Additive model

$growth = intercept + temperature + nitrogen$

$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_j x_{ij} + \epsilon_i$

Y	X1	X2
3	22.7	0.9
2.5	23.7	0.5
6	25.7	0.6
5.5	29.1	0.7
9	22	0.8
8.6	29	1.3
12	29.4	1

1.4. Multiple Linear Regression

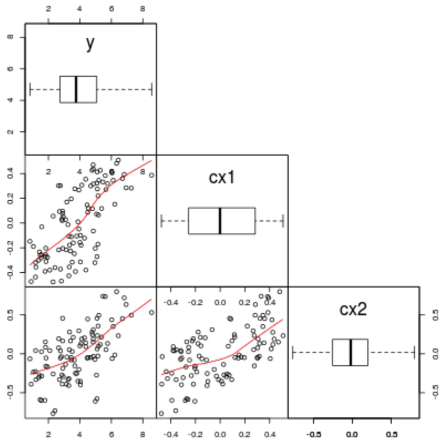
1.4.1. Multiplicative model

$$growth = intercept + temp + nitro + temp \times nitro$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2} + ... + \epsilon_i$$

1.5. Assumtions

- normality, homogeneity of variance, linearity
- (multi)collinearity



1.6. Multiple Linear Regression

1.6.1. Variance inflation

$$var.inf = \frac{1}{1 - R^2}$$

Collinear when $var.inf >= 5$

Some prefer > 3

2. Worked Examples

2.1. Worked examples

```
loyn <- read.csv('../data/loyn.csv', strip.white=T)
head(loyn)
```

```
  ABUND AREA YR.ISOL DIST LDIST GRAZE ALT
1   5.3  0.1    1968   39    39     2 160
2   2.0  0.5    1920  234   234     5  60
3   1.5  0.5    1900  104   311     5 140
4  17.1  1.0    1966   66    66     3 160
5  13.8  1.0    1918  246   246     5 140
6  14.1  1.0    1965  234   285     3 130
```

2.2. Worked Examples

Question: what effects do fragmentation variables have on the abundance of forest birds

Linear model:

$$Abund_i \sim \mathcal{N}(\mu, \sigma^2)$$
$$\mu = \beta_0 + \sum_{j=1:n}^{N} \beta_j X_{ji}$$
$$\beta_0, \beta_j \sim \mathcal{N}(0, 1000)$$
$$\sigma \sim Cauchy(0, 5)$$

2.3. Worked Examples

```
paruelo <- read.csv('../data/paruelo.csv', strip.white=T)
head(paruelo)
```

```
    C3   LAT   LONG MAP  MAT JJAMAP DJFMAP
1 0.65 46.40 119.55 199 12.4   0.12   0.45
2 0.65 47.32 114.27 469  7.5   0.24   0.29
3 0.76 45.78 110.78 536  7.2   0.24   0.20
4 0.75 43.95 101.87 476  8.2   0.35   0.15
5 0.33 46.90 102.82 484  4.8   0.40   0.14
6 0.03 38.87  99.38 623 12.0   0.40   0.11
```

2.4. Worked Examples

Question: what effects do fragmentation geographical variables have on the abundance of C3 grasses

Linear model:

$$
\begin{aligned}
\sqrt{C3_i} &\sim \mathcal{N}(\mu, \sigma^2) \\
\mu &= \beta_0 + \sum_{j=1:n}^{N} \beta_j X_{ji} \\
\beta_0, \beta_j &\sim \mathcal{N}(0, 1000) \\
\sigma &\sim Cauchy(0, 5)
\end{aligned}
$$