Workshop 7.6a: Factorial ANOVA

Murray Logan

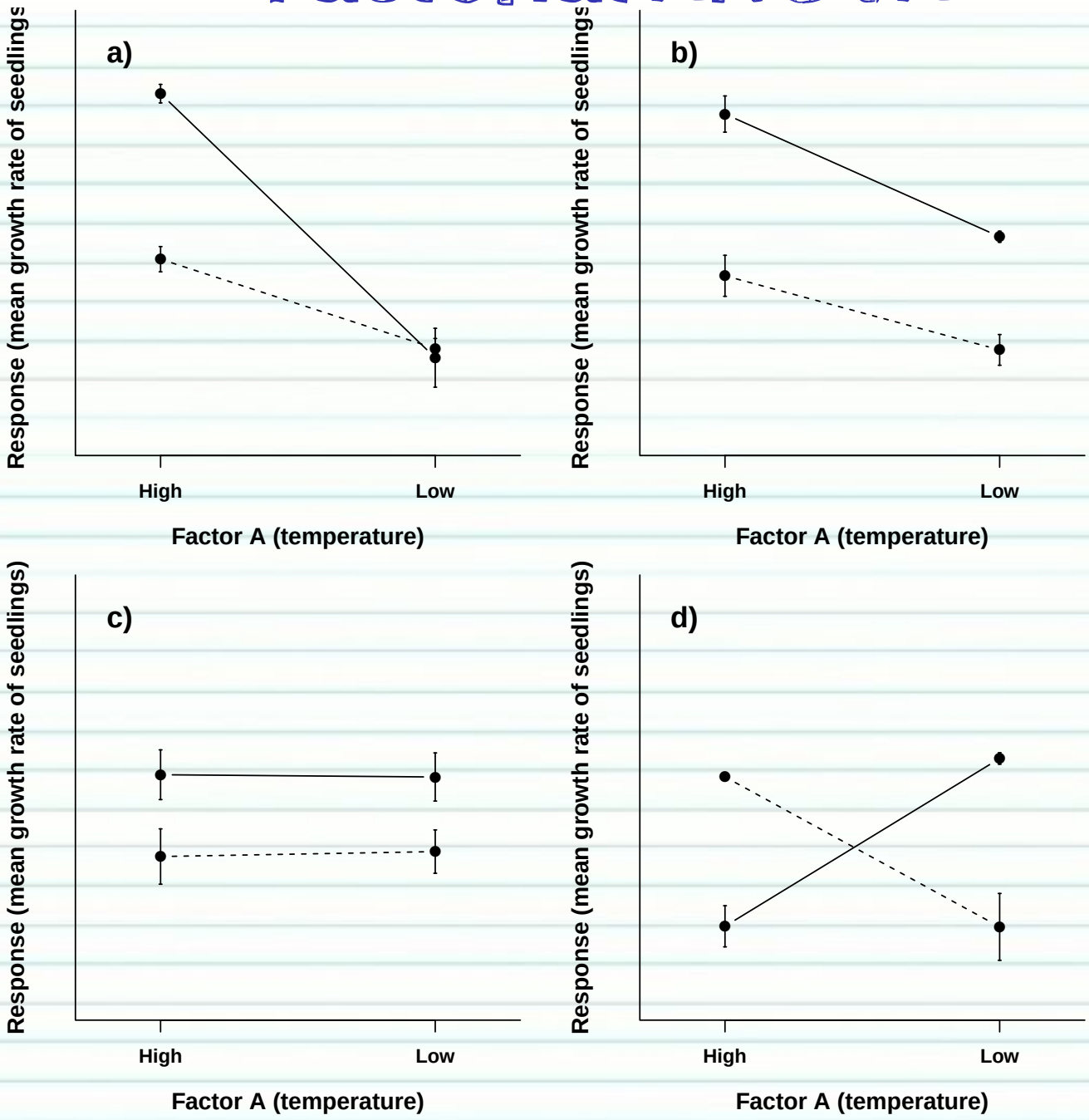
19 Jul 2017

Section 1

Background

Factorial ANOVA

Factorial ANOVA



The linear model

Two-factor

	Low N	Medium N	High N
Low temp.	XXX	XXX	XXX
High temp	XXX	XXX	XXX

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk}$$

- α_i is the effect of the i_{th} temperature
- β_j is the effect of the j_{th} nitrogen level
- $\alpha_i\beta_j$ is the effect of the ij_{th} interaction.

The linear model

Two-factor

	Low N	Medium N	High N
Low temp.	XXX	XXX	XXX
High temp	XXX	XXX	XXX

Temp	Nitrogen
Low	Low
Low	Low
Low	Low
Low	Medium
Low	Medium
Low	Medium
Low	High
Low	High
Low	High
High	Low
High	Low

The linear model

Two-factor

T	N	NA	(Intercept)	THigh	NMedium	NHigh	THigh:NMedium	THigh:NHigh
Low	Low	NA	1	0	0	0	0	0
Low	Low	NA	1	0	0	0	0	0
Low	Low	NA	1	0	0	0	0	0
Low	Medium	NA	1	0	1	0	0	0
Low	Medium	NA	1	0	1	0	0	0
Low	Medium	NA	1	0	1	0	0	0
Low	High	NA	1	0	0	1	0	0
Low	High	NA	1	0	0	1	0	0
Low	High	NA	1	0	0	1	0	0
High	Low	NA	1	1	0	0	0	0
High	Low	NA	1	1	0	0	0	0
High	Low	NA	1	1	0	0	0	0
High	Medium	NA	1	1	1	0	1	0
High	Medium	NA	1	1	1	0	1	0
High	Medium	NA	1	1	1	0	1	0
High	High	NA	1	1	0	1	0	1
High	High	NA	1	1	0	1	0	1
High	High	NA	1	1	0	1	0	1

The linear model

Two-factor

	Low N	Medium N	High N
Low temp.	XXX	XXX	XXX
High temp	XXX	XXX	XXX

$$y_i = \beta_{0i} + \beta_{1i} + \beta_{2i} + \beta_{3i} + \beta_{4i} + \beta_{5i} + \beta_{6i} + \varepsilon i$$

- β_0 is the mean of the $T_L : N_L$ group
- β_1 is the difference between $T_H : N_L$ and $T_L : N_L$
- β_2 is the difference between $T_L : N_M$ and

The linear model

Two-factor

	Low N	Medium N	High N
Low temp.	XXX	XXX	XXX
High temp	XXX	XXX	XXX

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk}$$

- α_i is the effect of the i_{th} temperature at the base level of β

?

Factorial ANOVA

Factor	MS	F-ratio (both fixed)	F-ratio (A fixed, B random)	F-ratio (both random)
A	MS_A	MS_A / MS_{Resid}	$MS_A / MS_{A:B}$	$MS_A / MS_{A:B}$
B	MS_B	MS_B / MS_{Resid}	MS_B / MS_{Resid}	$MS_B / MS_{A:B}$
A:B	$MS_{A:B}$	$MS_{A:B} / MS_{Resid}$	$MS_{A:B} / MS_{Resid}$	$MS_{A:B} / MS_{Resid}$

Section 2

Design Balance

Balance

When balanced

$$SS_{TOTAL} = SS_A + SS_B + SS_{A:B} + SS_{Resid}$$

Factoral ANOVA

DESIGN BALANCE

- When balanced

$$SS_{TOTAL} = SS_A + SS_B + SS_{A:B} + SS_{Resid}$$

- When not balanced

$$SS_{TOTAL} \neq SS_A + SS_B + SS_{A:B} + SS_{Resid}$$

Factorial ANOVA

Factoral ANOVA

DESIGN BALANCE

- When balanced

$$SS_{TOTAL} = SS_A + SS_B + SS_{A:B} + SS_{Resid}$$

- When not balanced

$$SS_{TOTAL} \neq SS_A + SS_B + SS_{A:B} + SS_{Resid}$$

- can't use sequential SS (Type I SS)

Factorial ANOVA

Factorial ANOVA

ASSUMPTIONS

- Normality
- Homogeneity of variance
- Independence
- Considerations for Balance

Section 3

Worked examples

Worked examples

```
> #Worked examples
> stehman <- read.csv('../data/stehman.csv', strip.white=T)
```

```
Error in file(file, "rt"): cannot open the connection
```

```
> head(stehman)
```

```
Error in head(stehman): object 'stehman' not found
```

Worked Examples

Question: what effects do pH and health have on the bud emergence rating of spruce seedlings

Linear model:

$$\text{Buds}_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk} \qquad \varepsilon \sim \mathcal{N}(0, \sigma^2)$$

Worked Examples

```
Error in file(file, "rt"): cannot open the connection

Error in head(quinn): object 'quinn' not found
```

Worked Examples

Question: what effects do season and density have on barnacle recruitment

Linear model:

$$\text{Recruits}_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk} \qquad \varepsilon \sim \mathcal{N}(0, \sigma^2)$$