

# Workshop 7.6b: Factorial ANOVA (Bayesian)

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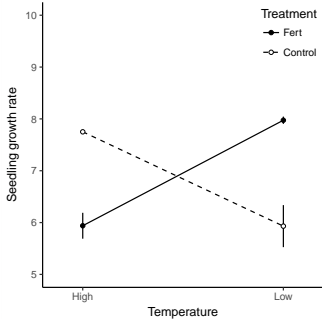
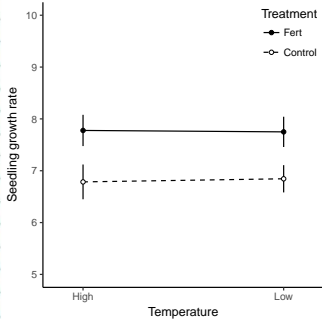
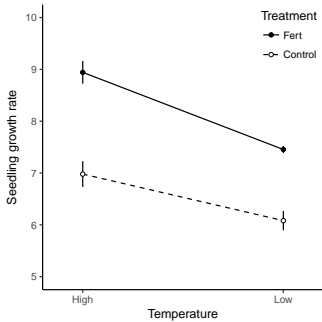
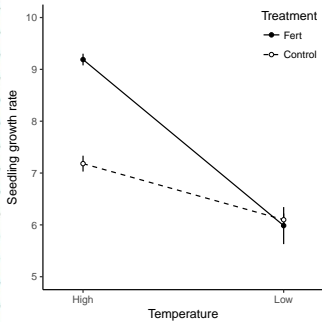
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# Section 1

## Background

# Factorial ANOVA

# Factorial ANOVA



# The linear model

Two-factor

|           | Low N | Medium N | High N |
|-----------|-------|----------|--------|
| Low temp. | XXX   | XXX      | XXX    |
| High temp | XXX   | XXX      | XXX    |

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk}$$

- $\alpha_i$  is the effect of the  $i_{th}$  temperature
- $\beta_j$  is the effect of the  $j_{th}$  nitrogen level
- $\alpha_i\beta_j$  is the effect of the  $ij_{th}$  interaction.

# The linear model

Two-factor

|           | Low N                              | Medium N  | High N  |
|-----------|------------------------------------|---|---|
| Low temp. | XXX ( $\eta = \mu$ )               | XXX ( $\eta = \mu - \beta_2$ )                                | XXX ( $\eta = \mu - \beta_3$ )                                |
| High temp | XXX<br>( $\eta = \mu - \alpha_2$ ) | XXX ( $\eta =$<br>$\mu - \alpha_2 - \beta_2 - \gamma_{2,2}$ ) | XXX ( $\eta =$<br>$\mu - \alpha_2 - \beta_3 - \gamma_{2,3}$ ) |

$$y_{ijk} \sim N(\eta_{ijk}, \sigma^2)$$

$$\eta_{ijk} = \mu + \sum_{j=2}^J \alpha_j T_j + \sum_{k=2}^K \beta_k N_k + \sum_{j=2, k=2}^{J \times K} \gamma_{jk} TN_{jk}$$

# The linear model

Two-factor

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|           | Low N | Medium N | High N |
|-----------|-------|----------|--------|
| Low temp. | XXX   | XXX      | XXX    |
| High temp | XXX   | XXX      | XXX    |

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Temp      Nitrogen

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|      |        |
|------|--------|
| Low  | Low    |
| Low  | Low    |
| Low  | Low    |
| Low  | Medium |
| Low  | Medium |
| Low  | Medium |
| Low  | High   |
| Low  | High   |
| Low  | High   |
| High | Low    |
| High | Low    |

# The linear model

Two-factor

| T    | N      | NA | (Intercept) | THigh | NMedium | NHigh | THigh:NMedium | THigh:NHigh |
|------|--------|----|-------------|-------|---------|-------|---------------|-------------|
| Low  | Low    | NA | 1           | 0     | 0       | 0     | 0             | 0           |
| Low  | Low    | NA | 1           | 0     | 0       | 0     | 0             | 0           |
| Low  | Low    | NA | 1           | 0     | 0       | 0     | 0             | 0           |
| Low  | Medium | NA | 1           | 0     | 1       | 0     | 0             | 0           |
| Low  | Medium | NA | 1           | 0     | 1       | 0     | 0             | 0           |
| Low  | Medium | NA | 1           | 0     | 1       | 0     | 0             | 0           |
| Low  | High   | NA | 1           | 0     | 0       | 1     | 0             | 0           |
| Low  | High   | NA | 1           | 0     | 0       | 1     | 0             | 0           |
| Low  | High   | NA | 1           | 0     | 0       | 1     | 0             | 0           |
| High | Low    | NA | 1           | 1     | 0       | 0     | 0             | 0           |
| High | Low    | NA | 1           | 1     | 0       | 0     | 0             | 0           |
| High | Low    | NA | 1           | 1     | 0       | 0     | 0             | 0           |
| High | Medium | NA | 1           | 1     | 1       | 0     | 1             | 0           |
| High | Medium | NA | 1           | 1     | 1       | 0     | 1             | 0           |
| High | Medium | NA | 1           | 1     | 1       | 0     | 1             | 0           |
| High | High   | NA | 1           | 1     | 0       | 1     | 0             | 1           |
| High | High   | NA | 1           | 1     | 0       | 1     | 0             | 1           |
| High | High   | NA | 1           | 1     | 0       | 1     | 0             | 1           |



# (Bayesian) linear model

Two-factor

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|           | Low N                             | Medium N   | High N   |
|-----------|-----------------------------------|--|--|
| Low temp. | Low ( $\eta = \alpha$ )           | Low ( $\eta = \alpha - \beta_3$ )                          | ( $\eta = \alpha - \beta_4$ )                          |
| High temp | Low ( $\eta = \alpha - \beta_2$ ) | Low ( $\eta =$<br>$\alpha - \beta_2 - \beta_3 - \beta_5$ ) | ( $\eta =$<br>$\alpha - \beta_2 - \beta_4 - \beta_6$ ) |

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$$y_{ij} \sim N(\eta_{ij}, \sigma^2)$$

$$\eta_{ij} = \alpha + \beta X$$

$$\alpha \sim N(0, 1000)$$

$$\beta_j \sim N(0, 1000)$$

$$\sigma^2 \sim \text{cauchy}(0, 4)$$

# Factorial ANOVA

## ASSUMPTIONS

- Normality
- Homogeneity of variance
- Independence
- Considerations for Balance

## MCMC RELATED

- Chain mixing and convergence diagnostics

# Section 2

Worked  
examples

# Worked examples

```
Error in file(file, "rt"): cannot open the connection
```

```
Error in head(quinn): object 'quinn' not found
```

# Worked Examples

Question: what effects do season and density have on barnacle recruitment

Linear model:

$$\text{Recruits}_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk} \quad \varepsilon \sim \mathcal{N}(0, \sigma^2)$$