Workshop 7.6b: Factorial ANOVA (Bayesian)

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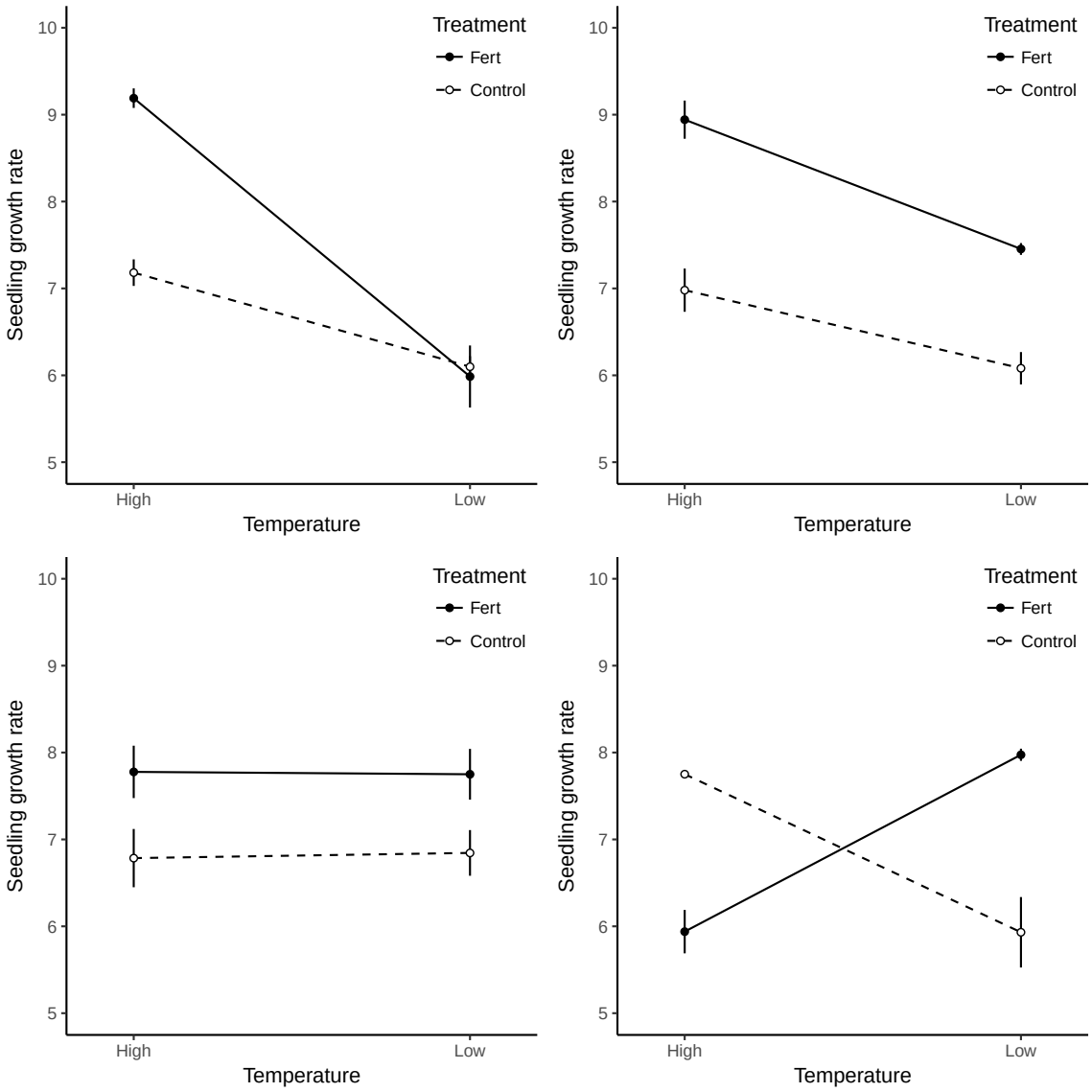
1. Background

1.1. Factorial ANOVA

1.2. Factorial ANOVA



1.3. The linear model

Two-factor

	Low N	Medium N	High N
Low temp.	XXX	XXX	XXX
High temp	XXX	XXX	XXX

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk}$$

- α_i is the effect of the i_{th} temperature
- β_j is the effect of the j_{th} nitrogen level
- $\alpha_i\beta_j$ is the effect of the ij_{th} interaction.

1.4. The linear model

Two-factor

	Low N	Medium N	High N
Low temp.	XXX ($\eta = \mu$)	XXX ($\eta = \mu - \beta_2$)	XXX ($\eta = \mu - \beta_3$)
High temp	XXX ($\eta = \mu - \alpha_2$)	XXX ($\eta = \mu - \alpha_2 - \beta_2 - \gamma_{2,2}$)	XXX ($\eta = \mu - \alpha_2 - \beta_3 - \gamma_{2,3}$)

$$
\begin{aligned}
y_{ijk} &\sim N(\eta_{ijk}, \sigma^2) \\
\eta_{ijk} &= \mu + \sum_{j=2}^{J} \alpha_j \mathbf{T}_j + \sum_{k=2}^{K} \beta_k \mathbf{N}_k \\
&\quad + \sum_{j=2,k=2}^{J \times K} \gamma_{jk} \mathbf{TN}_{jk}
\end{aligned}
$$

1.5. The linear model

Two-factor

	Low N	Medium N	High N
Low temp.	XXX	XXX	XXX
High temp	XXX	XXX	XXX

Temp	Nitrogen
Low	Low
Low	Low
Low	Low
Low	Medium
Low	Medium
Low	Medium
Low	High
Low	High
Low	High
High	Low
High	Low
High	Low
High	Medium
High	Medium
High	Medium
High	High
High	High
High	High

$$y_i = \beta_{0i} + \beta_{1i} + \beta_{2i} + \beta_{3i} + \beta_{4i} + \beta_{5i} + \beta_{6i} + \varepsilon i$$

1.6. The linear model

Two-factor

T	N	NA	(Intercept)	THigh	NMedium	NHigh	THigh:NMedium	THigh:NHigh
Low	Low	NA	1	0	0	0	0	0
Low	Low	NA	1	0	0	0	0	0
Low	Low	NA	1	0	0	0	0	0
Low	Medium	NA	1	0	1	0	0	0
Low	Medium	NA	1	0	1	0	0	0
Low	Medium	NA	1	0	1	0	0	0
Low	High	NA	1	0	0	1	0	0
Low	High	NA	1	0	0	1	0	0
Low	High	NA	1	0	0	1	0	0
High	Low	NA	1	1	0	0	0	0
High	Low	NA	1	1	0	0	0	0
High	Low	NA	1	1	0	0	0	0
High	Medium	NA	1	1	1	0	1	0
High	Medium	NA	1	1	1	0	1	0
High	Medium	NA	1	1	1	0	1	0
High	High	NA	1	1	0	1	0	1
High	High	NA	1	1	0	1	0	1
High	High	NA	1	1	0	1	0	1

1.7. (Bayesian) linear model

Two-factor

	Low N	Medium N	High N
Low temp.	Low ($\eta = \alpha$)	Low ($\eta = \alpha - \beta_3$)	($\eta = \alpha - \beta_4$)
High temp	Low ($\eta = \alpha - \beta_2$)	Low ($\eta = \alpha - \beta_2 - \beta_3 - \beta_5$)	($\eta = \alpha - \beta_2 - \beta_4 - \beta_6$)

$$y_{ij} \sim N(\eta_{ij}, \sigma^2) \eta_{ij} = \alpha + \beta X$$

$$\alpha \sim N(0, 1000)$$
$$\beta_j \sim N(0, 1000)$$
$$\sigma^2 \sim cauchy(0, 4)$$

1.8. Factorial ANOVA

1.8.1. Assumptions

- Normality
- Homogeneity of variance
- Independence
- Considerations for **Balance**

1.8.2. MCMC related

- Chain mixing and convergence diagnostics

2. Worked examples

2.1. Worked examples

```
Error in file(file, "rt"): cannot open the connection
```

```
Error in head(quinn): object 'quinn' not found
```

2.2. Worked Examples

Question: what effects do season and density have on barnacle recruitment

Linear model:

$$Recruits_{ijk} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{ijk} \qquad \varepsilon \sim \mathcal{N}(0, \sigma^2)$$