Workshop 8.3a: Non-independence part 1

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Section 1

Linear modelling assumptions

Linear modelling assumptions

$$y_i = \beta_0 + \beta_1 \times x_i + \varepsilon_i$$

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

Homogeneity of variance

$$y_i = \underbrace{\beta_0 + \beta_1 \times x_i}_{\text{Linearity}} + \varepsilon_i \qquad \varepsilon_i \sim \underbrace{\mathcal{N}(0, \sigma^2)}_{\text{Normality}} \qquad \mathbf{V} = cov = \begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & \vdots \\ \vdots & \cdots & \sigma^2 & \vdots \\ 0 & \cdots & \cdots & \sigma^2 \end{pmatrix}$$

Zero covariance (=independence)

Variance-covariance

$$\mathtt{V} = \underbrace{\begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & \vdots \\ \vdots & \cdots & \sigma^2 & \vdots \\ 0 & \cdots & \cdots & \sigma^2 \end{pmatrix}}_{\text{Variance-covariance matrix}}$$

Compound symmetry

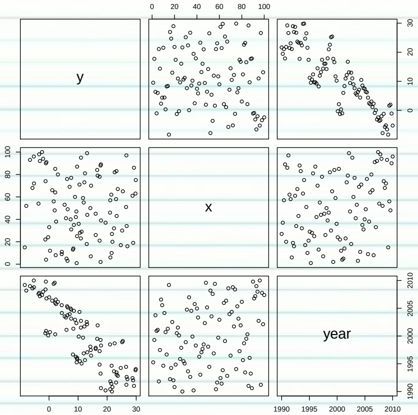
- constant correlation (and cov)
- sphericity

$$\text{cor}(\varepsilon) = \underbrace{\begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \vdots \\ \ldots & \cdots & 1 & \vdots \\ \rho & \cdots & \cdots & 1 \end{pmatrix}}_{\text{Correlation matrix}}$$

$$\text{V} = \underbrace{\begin{pmatrix} \theta + \sigma^2 & \theta & \cdots & \theta \\ \theta & \theta + \sigma^2 & \cdots & \vdots \\ \vdots & \cdots & \theta + \sigma^2 & \vdots \\ \theta & \cdots & \cdots & \theta + \sigma^2 \end{pmatrix}}_{\text{Variance-covariance matrix}}$$

Temporal autocorrelation

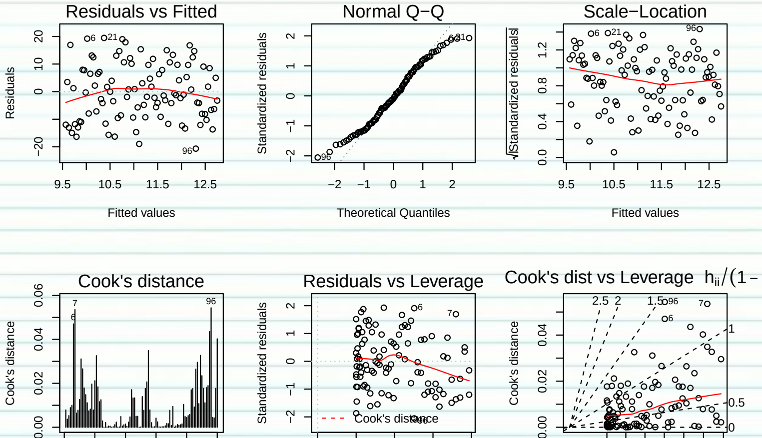
- correlation dependent on proximity
- data.t



Temporal autocorrelation

- Relationship between Y and X

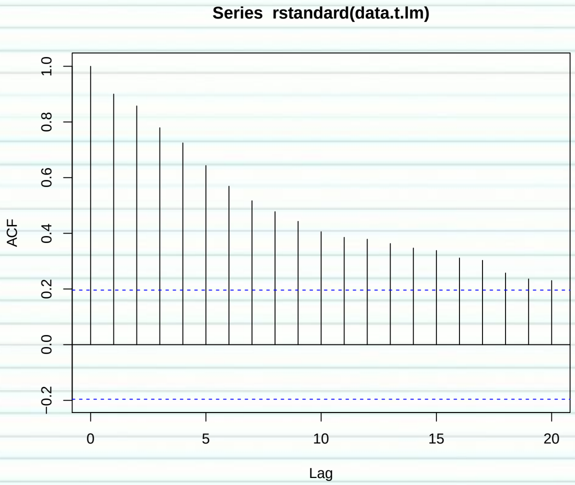
```
> data.t.lm <- lm(y~x, data=data.t)
> par(mfrow=c(2,3))
> plot(data.t.lm, which=1:6, ask=FALSE)
```



Temporal autocorrelation

- Relationship between Y and X

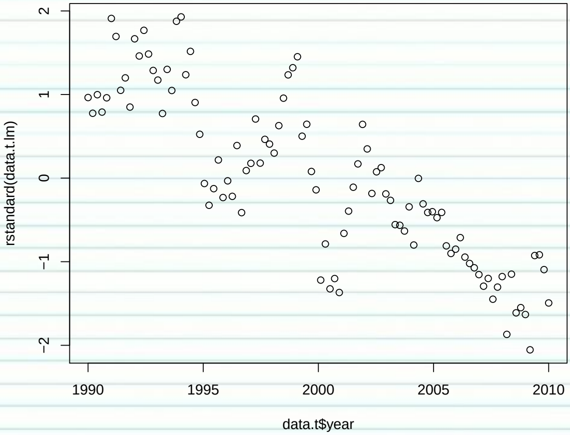
```
> acf(rstandard(data.t.lm))
```



Temporal autocorrelation

- can we partial out time

```
> plot(rstandard(data.t.lm)~data.t$year)
```



Temporal autocorrelation

- can we partial out time

```
> library(car)
> vif(lm(y~x+year, data=data.t))
```

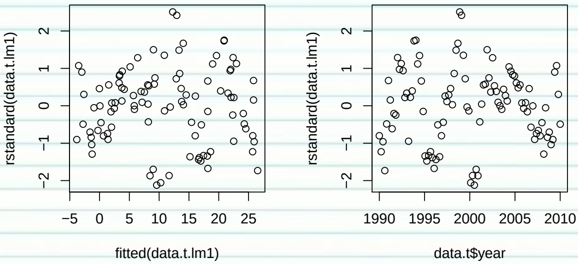
```
       x     year
1.040037 1.040037
```

```
> data.t.lm1 <- lm(y~x+year, data.t)
```

Testing for autocorrelation

RESIDUAL PLOT

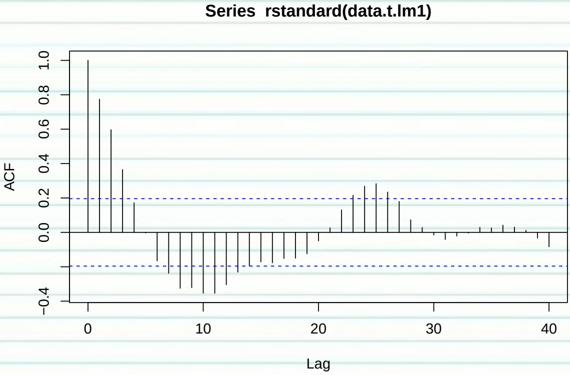
```
> par(mfrow=c(1,2))
> plot(rstandard(data.t.lm1)~fitted(data.t.lm1))
> plot(rstandard(data.t.lm1)~data.t$year)
```



Testing for autocorrelation

AUTOCORRELATION (ACF) PLOT

```
> acf(rstandard(data.t.lm1), lag=40)
```



First order autocorrelation (AR1)

$$\text{cor}(\varepsilon) = \underbrace{\begin{pmatrix} 1 & \rho & \cdots & \rho^{|t-s|} \\ \rho & 1 & \cdots & \vdots \\ \vdots & \cdots & 1 & \vdots \\ \rho^{|t-s|} & \cdots & \cdots & 1 \end{pmatrix}}_{\text{First order autoregressive correlation structure}}$$

where:

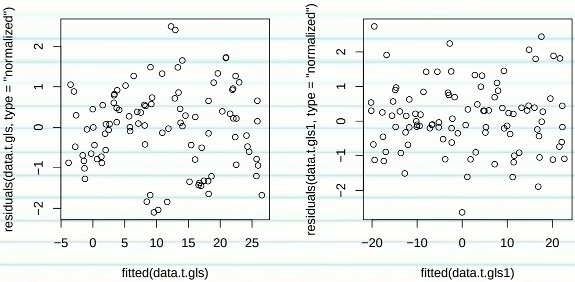
- s and t are the times.
- $s - t$ is the lag

First order auto-regressive (AR1)

```
> library(nlme)
> data.t.gls <- gls(y~x+year, data=data.t, method='REML')
> data.t.gls1 <- gls(y~x+year, data=data.t,
+          correlation=corAR1(form=~year),method='REML')
```

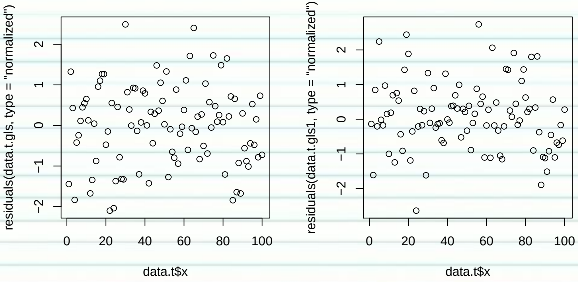
First order auto-regressive (AR1)

```
> par(mfrow=c(1,2))
> plot(residuals(data.t.gls, type="normalized")~
+      fitted(data.t.gls))
> plot(residuals(data.t.gls1, type="normalized")~
+      fitted(data.t.gls1))
```



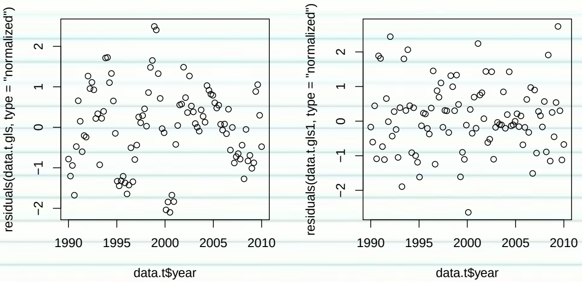
First order auto-regressive (AR1)

```
> par(mfrow=c(1,2))
> plot(residuals(data.t.gls, type="normalized")~
+        data.t$x)
> plot(residuals(data.t.gls1, type="normalized")~
+        data.t$x)
```



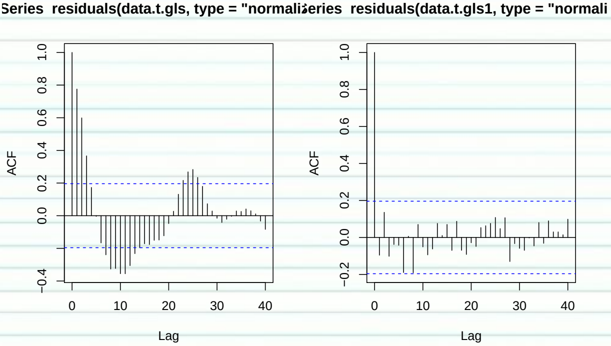
First order auto-regressive (AR1)

```
> par(mfrow=c(1,2))
> plot(residuals(data.t.gls, type="normalized")~
+       data.t$year)
> plot(residuals(data.t.gls1, type="normalized")~
+       data.t$year)
```



First order auto-regressive (AR1)

```
> par(mfrow=c(1,2))
> acf(residuals(data.t.gls, type='normalized'), lag=40)
> acf(residuals(data.t.gls1, type='normalized'), lag=40)
```



First order auto-regressive (AR1)

```
> AIC(data.t.gls, data.t.gls1)
```

```
            df      AIC
data.t.gls   4 626.3283
data.t.gls1  5 536.7467
```

```
> anova(data.t.gls, data.t.gls1)
```

```
            Model df      AIC      BIC    logLik   Test  L.Ratio p-value
data.t.gls      1  4 626.3283 636.6271 -309.1642
data.t.gls1     2  5 536.7467 549.6203 -263.3734 1 vs 2 91.58158  <.0001
```

Auto-regressive moving average (ARMA)

```
> data.t.gls2 <- update(data.t.gls,
+     correlation=corARMA(form=~year,p=2,q=0))
> data.t.gls3 <- update(data.t.gls,
+     correlation=corARMA(form=~year,p=3,q=0))
> AIC(data.t.gls, data.t.gls1, data.t.gls2, data.t.gls3)
```

```
            df      AIC
data.t.gls   4 626.3283
data.t.gls1  5 536.7467
data.t.gls2  6 538.1032
data.t.gls3  7 538.8376
```

Summarize model

```
> summary(data.t.gls1)
```

```
Generalized least squares fit by REML
  Model: y ~ x + year
  Data: data.t
       AIC       BIC    logLik
  536.7467 549.6203 -263.3734

Correlation Structure: ARMA(1,0)
 Formula: ~year
 Parameter estimate(s):
     Phi1
0.9126603

Coefficients:
               Value Std.Error   t-value p-value
(Intercept) 4388.568 1232.6129  3.560378  0.0006
x              0.028    0.0086  3.296648  0.0014
year          -2.195    0.6189 -3.545955  0.0006

 Correlation:
     (Intr) x
x     0.009
year -1.000 -0.010
```

Section 2

Spatial auto-correlation

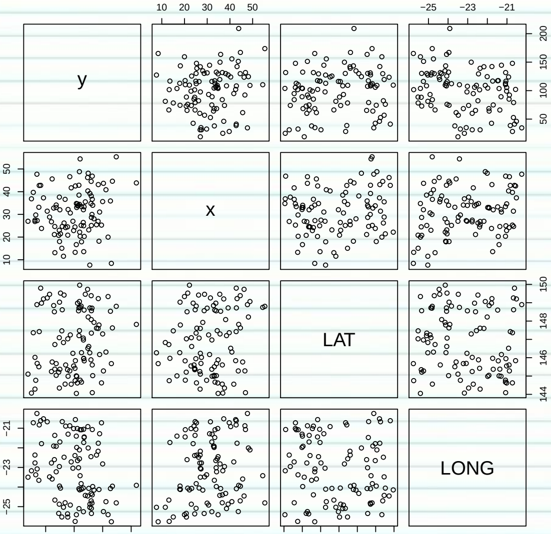
Spatial autocorrelation

- similar, yet dependency is 2d
- 2d Euclidean dissimilarity
- Exponential decay

$$\mathrm{cor}(\varepsilon) = \underbrace{\begin{pmatrix} 1 & e^{-\delta} & \cdots & e^{-\delta D} \\ e^{-\delta} & 1 & \cdots & \vdots \\ \vdots & \cdots & 1 & \vdots \\ e^{-\delta D} & \cdots & \cdots & 1 \end{pmatrix}}_{\text{Exponential autoregressive correlation structure}}$$

Spatial autocorrelation

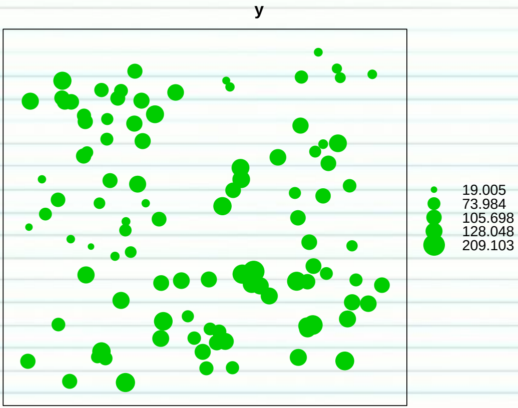
- data.s



Spatial autocorrelation

- Spatial arrangement of Y

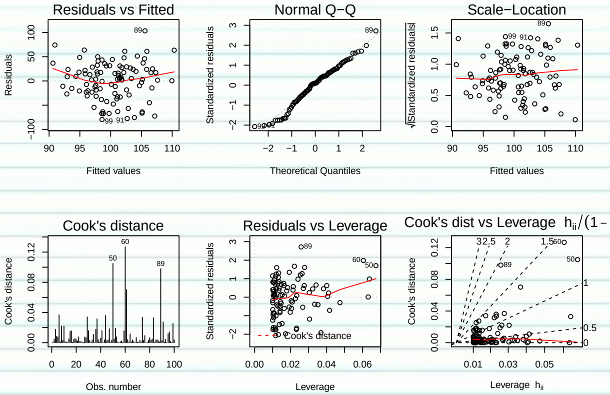
```
> library(sp)
> coordinates(data.s) <- ~LAT+LONG
> bubble(data.s,'y')
```



Spatial autocorrelation

- Relationship between Y and X

```
> data.s.lm <- lm(y~x, data=data.s)
> par(mfrow=c(2,3))
> plot(data.s.lm, which=1:6, ask=FALSE)
```

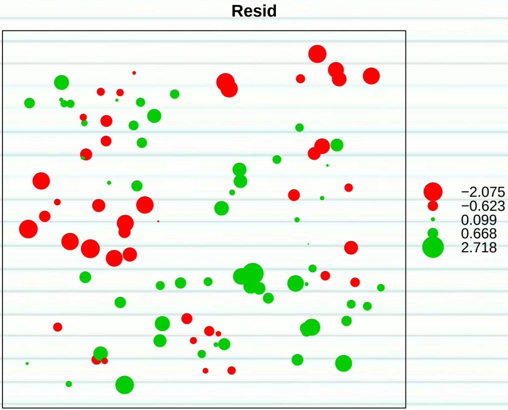


Detecting spatial autocorrelation

- bubble plot
- semi-variogram

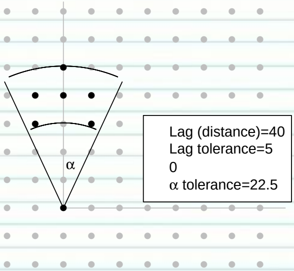
Bubble plot

```
> data.s$Resid <- rstandard(data.s.lm)
> library(sp)
> #coordinates(data.s) <- ~LAT+LONG
> bubble(data.s,'Resid')
```

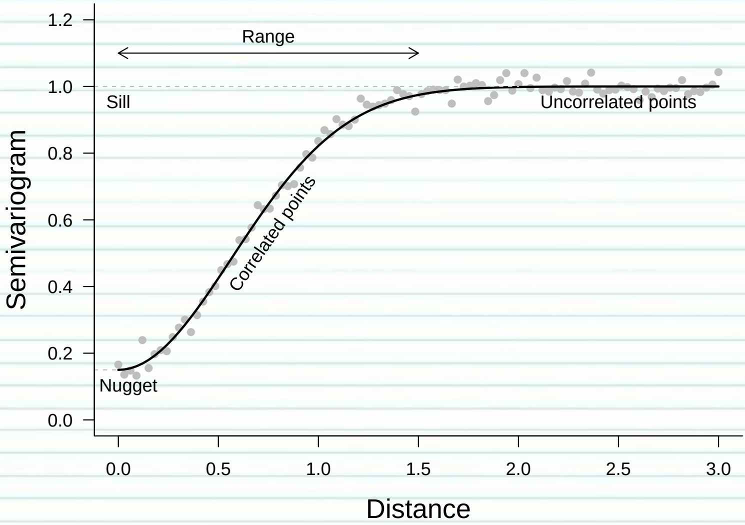


Semi-variogram

- semivariance = similarity (of residuals) between pairs at specific distances
- distances �binned� according to distance and orientation (N)

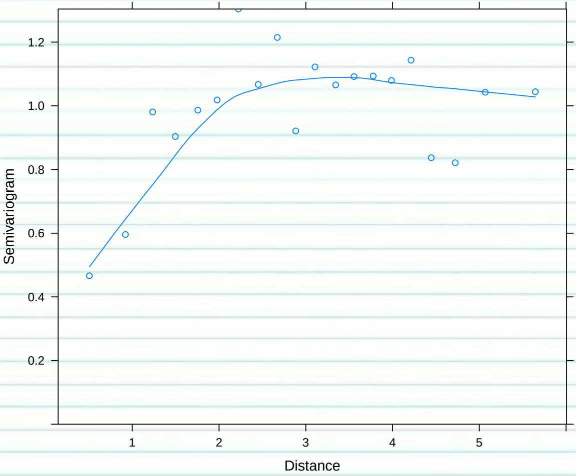


Semi-variogram



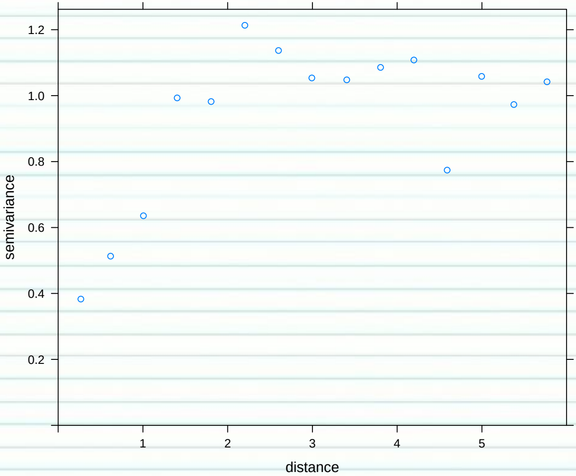
Semi-variogram

```
> library(nlme)
> data.s.gls <- gls(y~x, data.s, method='REML')
> plot(nlme::Variogram(data.s.gls, form=~LAT+LONG,
+         resType="normalized"))
```



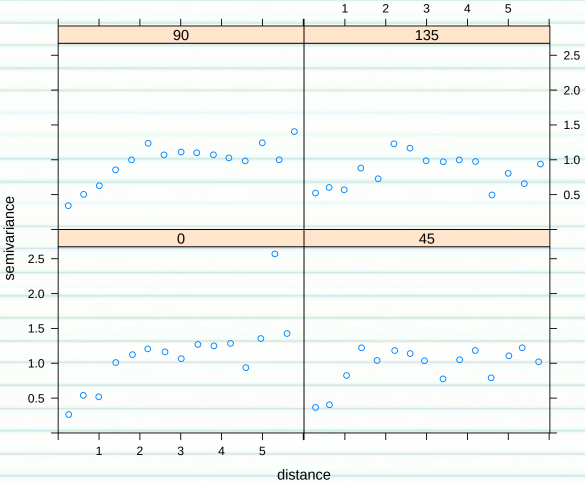
Semi-variogram

```
> library(gstat)
> plot(variogram(residuals(data.s.gls,"normalized")~1,
+        data=data.s, cutoff=6))
```



Semi-variogram

```
> library(gstat)
> plot(variogram(residuals(data.s.gls,"normalized")~1,
+     data=data.s, cutoff=6,alpha=c(0,45,90,135)))
```



Accommodating spatial autocorrelation

Correlation function	Correlation structure	Description
`corExp(form= lat+long)`	Exponential	$\Phi = 1 - e^{-D/\rho}$
`varGaus(form= lat+long)`	Gaussian	$\Phi = 1 - e^{-(D/\rho)^2}$
`varLin(form= lat+long)`	Linear	$\Phi = 1 - (1 - D/\rho)I(d < \rho)$
`varRatio(form= lat+long)`	Rational quadratic	$\Phi = (d/\rho)^2/(1 + (d/\rho)2)$
`varSpher(form= lat+long)`	Spherical	$\Phi = 1 - (1 - 1.5(d/\rho) +$

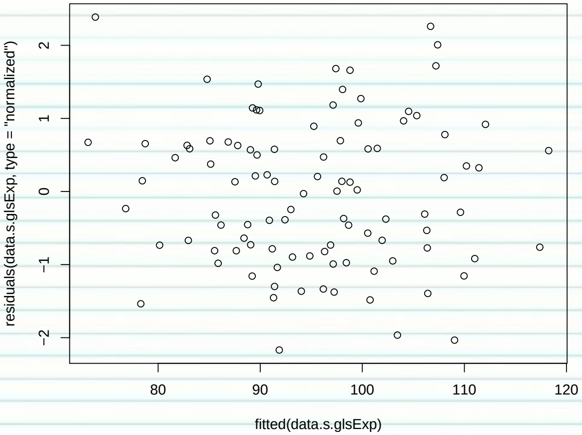
Accommodating spatial autocorrelation

```
> data.s.glsExp <- update(data.s.gls,
+    correlation=corExp(form=~LAT+LONG, nugget=TRUE))
> data.s.glsGaus <- update(data.s.gls,
+   correlation=corGaus(form=~LAT+LONG, nugget=TRUE))
> #data.s.glsLin <- update(data.s/gls,
> #  correlation=corLin(form=~LAT+LONG, nugget=TRUE))
> data.s.glsRatio <- update(data.s.gls,
+   correlation=corRatio(form=~LAT+LONG, nugget=TRUE))
> data.s.glsSpher <- update(data.s.gls,
+   correlation=corSpher(form=~LAT+LONG, nugget=TRUE))
>
> AIC(data.s.gls, data.s.glsExp, data.s.glsGaus, data.s.glsRatio, data.s.gl
```

```
                df       AIC
data.s.gls       3 1013.9439
data.s.glsExp    5  974.3235
data.s.glsGaus   5  976.4422
data.s.glsRatio  5  974.7862
data.s.glsSpher  5  975.5244
```

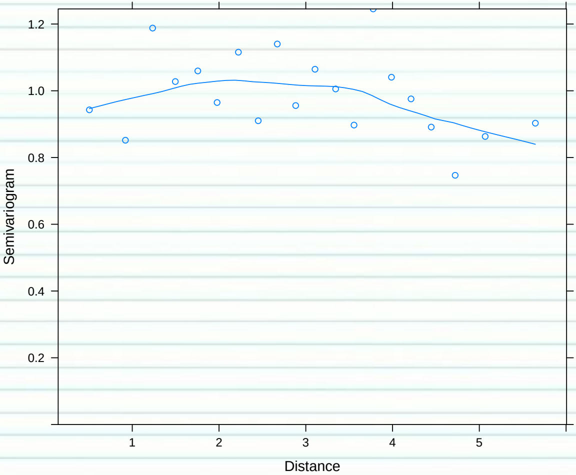
Accommodating spatial autocorrelation

```
> plot(residuals(data.s.glsExp, type="normalized")~
+      fitted(data.s.glsExp))
```



Accommodating spatial autocorrelation

```
> plot(nlme::Variogram(data.s.glsExp, form=~LAT+LONG,
+       resType="normalized"))
```



Summarize model

```
> summary(data.s.glsExp)

Generalized least squares fit by REML
  Model: y ~ x
  Data: data.s
       AIC      BIC    logLik
  974.3235 987.2484 -482.1618

Correlation Structure: Exponential spatial correlation
 Formula: ~LAT + LONG
 Parameter estimate(s):
    range    nugget
1.6956723 0.1280655

Coefficients:
               Value Std.Error  t-value p-value
(Intercept) 65.90018 21.824752 3.019516  0.0032
x            0.94572  0.286245 3.303886  0.0013

 Correlation:
  (Intr)
x -0.418

Standardized residuals:
       Min         Q1        Med         Q3        Max
```

Summarize model

```
> xs <- seq(min(data.s$x), max(data.s$x), l=100)
> xmat <- model.matrix(~x, data.frame(x=xs))
>
> mpred <- function(model, xmat, data.s) {
+   pred <- as.vector(coef(model) %*% t(xmat))
+   (se<-sqrt(diag(xmat %*% vcov(model) %*% t(xmat))))
+   ci <- data.frame(pred+outer(se,qt(df=nrow(data.s)-2,c(.025,.975))))
+   colnames(ci) <- c('lwr','upr')
+   data.s.sum<-data.frame(pred, x=xs,se,ci)
+   data.s.sum
+ }
>
> data.s.sum<-mpred(data.s.glsExp, xmat, data.s)
> data.s.sum$resid <- data.s.sum$pred+residuals(data.s.glsExp)
>
> library(ggplot2)
> ggplot(data.s.sum, aes(y=pred, x=x))+
+     geom_ribbon(aes(ymin=lwr, ymax=upr), fill='blue', alpha=0.2) +
+     geom_line()+geom_point(aes(y=resid)) + theme_classic()
```

Summarize model