

# Workshop 9.3a: Randomized block designs

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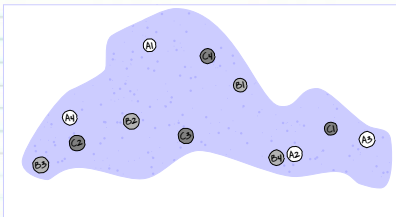
23 Nov 2016

# Section 1

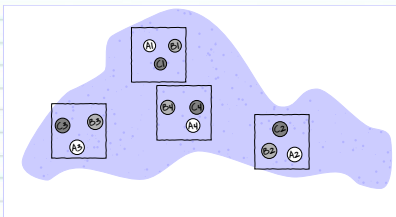
Randomized  
Block (RCB)  
designs

# RCB design

Simple



Randomized block design



# RCB design

$$y_{ij} = \mu + \beta_i + \alpha_j + \varepsilon_{ij}$$

$\mu$  - the mean of the first treatment group

$\beta$  - the random (Block) effect

$\alpha$  - the main within Block effect

e.g.

abundance = base + block + treatment

# Repeated measures designs

Subject 1

T0	T5	T10	20	T50	T100	T200	T500	T1000
----	----	-----	----	-----	------	------	------	-------

Subject 2

T0	T5	T10	20	T50	T100	T200	T500	T1000
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Subject 3

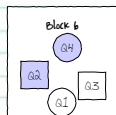
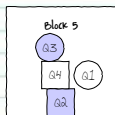
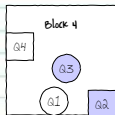
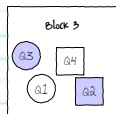
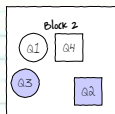
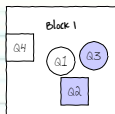
T0	T5	T10	20	T50	T100	T200	T500	T1000
----	----	-----	----	-----	------	------	------	-------

Subject 4

T0	T5	T10	20	T50	T100	T200	T500	T1000
----	----	-----	----	-----	------	------	------	-------

# Assumptions

- Normality, homogeneity of variance
- No Block by within-block interaction



Subject 1								
T0	T5	T10	T20	T50	T100	T200	T500	T1000

Subject 2								
T0	T5	T10	T20	T50	T100	T200	T500	T1000

Subject 3								
T0	T5	T10	T20	T50	T100	T200	T500	T1000

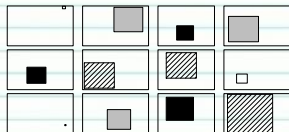
  

Subject 4								
T0	T5	T10	T20	T50	T100	T200	T500	T1000

# Assumptions

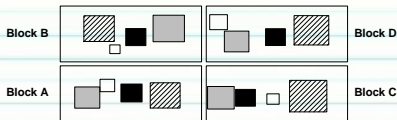
- Normality, homogeneity of variance
- No Block by within-block interaction
- Independence
  - (variance-covariance structure)

# Var-cov structure



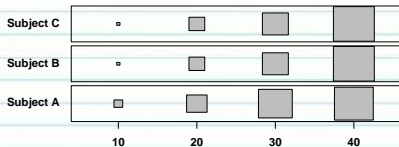
□ T1 ■ T2 ■ T3 ▨ T4

	T1	T2	T3	T4
T1	0.15	0.00	0.00	0.00
T2	0.00	0.15	0.00	0.00
T3	0.00	0.00	0.15	0.00
T4	0.00	0.00	0.00	0.15



□ T1 ■ T2 ■ T3 ▨ T4

	T1	T2	T3	T4
T1	0.15	0.05	0.05	0.05
T2	0.05	0.15	0.05	0.05
T3	0.05	0.05	0.15	0.05
T4	0.05	0.05	0.05	0.15



	T1	T2	T3	T4
T1	0.15	0.60	0.30	0.10
T2	0.60	0.15	0.60	0.30
T3	0.30	0.60	0.15	0.60
T4	0.10	0.30	0.60	0.15

Time (mins)



# Assumptions

- Normality, homogeneity of variance
- No Block by within-block interaction
- Independence
  - (variance-covariance structure)
    - RCB - usually met
    - Repeated measures - rarely met

# Example

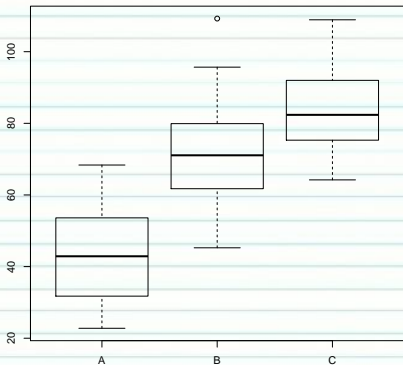
```
> data.rcb1 <- read.csv('../data/data.rcb1.csv', strip.white=TRUE)  
> head(data.rcb1)
```

	y	A	Block
1	37.39761	A	B1
2	61.47033	B	B1
3	78.07370	C	B1
4	30.59803	A	B2
5	59.00035	B	B2
6	76.72575	C	B2

# Exploratory data analysis

Normality and homogeneity of variance

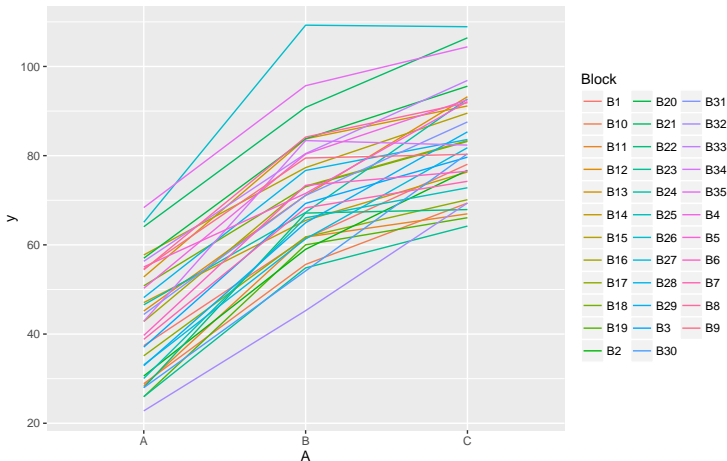
```
> boxplot(y~A, data.rcb1)
```



# Exploratory data analysis

No block by within-block interaction

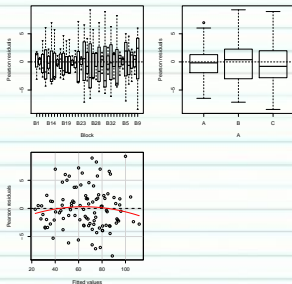
```
> library(ggplot2)
> ggplot(data.rcb1, aes(y=y, x=A, group=Block, color=Block)) +
+   geom_line() +
+   guides(color=guide_legend(ncol=3))
```



# Exploratory data analysis

No block by within-block interaction

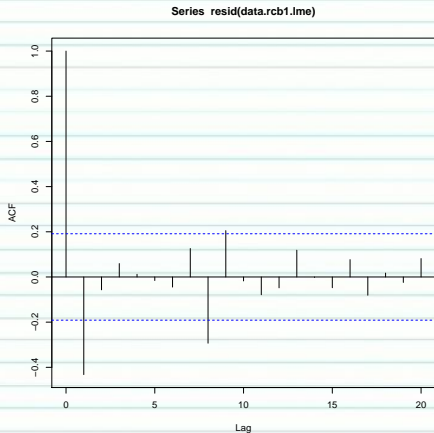
```
> library(car)  
> residualPlots(lm(y~Block+A, data.rcb1))
```



	Test stat	Pr(> t )
Block	NA	NA
A	NA	NA
Tukey test	-0.885	0.376

# Sphericity

```
> library(nlme)
> data.rcb1.lme <- lme(y~A, random=~1|Block,
+                       data=data.rcb1)
> acf(resid(data.rcb1.lme))
```



# Model fitting

```
> #Assuming sphericity
> data.rcb1.lme <- lme(y~A, random=~1|Block, data=data.rcb1,
+                      method='REML')
> data.rcb1.lme1 <- lme(y~A, random=~A|Block, data=data.rcb1,
+                       method='REML')
> AIC(data.rcb1.lme, data.rcb1.lme1)
```

	df	AIC
data.rcb1.lme	5	722.1087
data.rcb1.lme1	10	727.2001

```
> anova(data.rcb1.lme, data.rcb1.lme1)
```

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
data.rcb1.lme	1	5	722.1087	735.2336	-356.0544			
data.rcb1.lme1	2	10	727.2001	753.4499	-353.6001	1 vs 2	4.908574	0.4271

# Model fitting

```
> #Assuming sphericity
> data.rcb1.lme.AR1 <- lme(y~A, random=~1|Block, data=data.rcb1,
+ correlation=corAR1(),method='REML')
> data.rcb1.lme1.AR1 <- lme(y~A, random=~A|Block, data=data.rcb1,
+ correlation=corAR1(),method='REML')
> AIC(data.rcb1.lme, data.rcb1.lme1,data.rcb1.lme.AR1, data.rcb1.lme1.AR1)
```

	df	AIC
data.rcb1.lme	5	722.1087
data.rcb1.lme1	10	727.2001
data.rcb1.lme.AR1	6	723.3178
data.rcb1.lme1.AR1	11	729.2001

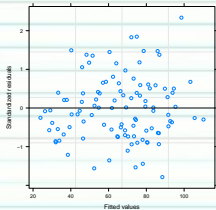
```
> anova(data.rcb1.lme, data.rcb1.lme1,data.rcb1.lme.AR1, data.rcb1.lme1.AR1)
```

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
data.rcb1.lme	1	5	722.1087	735.2336	-356.0544			
data.rcb1.lme1	2	10	727.2001	753.4499	-353.6001	1 vs 2	4.908574	0.4271
data.rcb1.lme.AR1	3	6	723.3178	739.0676	-355.6589	2 vs 3	4.117606	0.3903
data.rcb1.lme1.AR1	4	11	729.2001	758.0748	-353.6001	3 vs 4	4.117606	0.5326

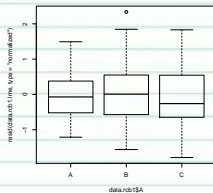


# Model validation

```
> plot(data.rcb1.lme)
```



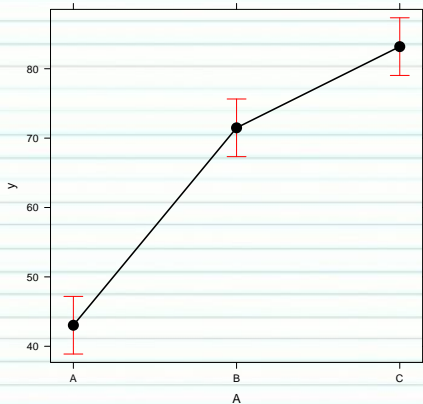
```
> plot(resid(data.rcb1.lme, type='normalized') ~  
+ data.rcb1$A)
```



# Effects plots

```
> library(effects)  
> plot(Effect('A', data.rcb1.lme))
```

A effect plot



# Parameter estimates

```
> summary(data.rcb1.lme)
```

Linear mixed-effects model fit by REML

Data: data.rcb1

	AIC	BIC	logLik
	722.1087	735.2336	-356.0544

Random effects:

Formula: ~1 | Block

(Intercept) Residual

StdDev: 11.51409 4.572284

Fixed effects: y ~ A

	Value	Std.Error	DF	t-value	p-value
(Intercept)	43.03434	2.094074	68	20.55053	0
AB	28.45241	1.092985	68	26.03185	0
AC	40.15556	1.092985	68	36.73936	0

Correlation:

(Intr) AB

AB -0.261

AC -0.261 0.500

Standardized Within-Group Residuals:

	Min	Q1	Med	Q3	Max
	-1.78748258	-0.57867597	-0.07108159	0.49990644	2.33727672

# Parameter estimates

```
> intervals(data.rcb1.lme)
```

Approximate 95% confidence intervals

Fixed effects:

	lower	est.	upper
(Intercept)	38.85568	43.03434	47.21300
AB	26.27140	28.45241	30.63343
AC	37.97455	40.15556	42.33658

attr("label")

[1] "Fixed effects:"

Random Effects:

Level: Block

	lower	est.	upper
sd((Intercept))	8.964236	11.51409	14.78925

Within-group standard error:

	lower	est.	upper
	3.864944	4.572284	5.409077

# Parameter estimates

```
> VarCorr(data.rcb1.lme)
```

```
Block = pdLogChol(1)
```

	Variance	StdDev
(Intercept)	132.57434	11.514093
Residual	20.90578	4.572284

# ANOVA table

```
> anova(data.rcb1.lme)
```

	numDF	denDF	F-value	p-value
(Intercept)	1	68	1089.3799	<.0001
A	2	68	714.0295	<.0001

$R^2$

[1] 0.6516126

[1] 0.300933

[1] 0.04745443

[1] 0.9525456

# What about lmer?

```
> library(lme4)
> data.rcb1.lmer <- lmer(y~A+(1|Block), data=data.rcb1, REML=TRUE,
+   control=lmerControl(check.nobs.vs.nRE="ignore"))
> data.rcb1.lmer1 <- lmer(y~A+(A|Block), data=data.rcb1, REML=TRUE,
+   control=lmerControl(check.nobs.vs.nRE="ignore"))
> AIC(data.rcb1.lmer, data.rcb1.lmer1)
```

	df	AIC
data.rcb1.lmer	5	722.1087
data.rcb1.lmer1	10	727.2001

```
> anova(data.rcb1.lmer, data.rcb1.lmer1)
```

Data: data.rcb1

Models:

data.rcb1.lmer: y ~ A + (1 | Block)

data.rcb1.lmer1: y ~ A + (A | Block)

	Df	AIC	BIC	logLik	deviance	Chisq	Chi Df	Pr(>Chisq)
data.rcb1.lmer	5	729.03	742.30	-359.51	719.03			
data.rcb1.lmer1	10	733.98	760.52	-356.99	713.98	5.0529	5	0.4095



# What about lmer?

```
> summary(data.rcb1.lmer)
```

```
Linear mixed model fit by REML ['lmerMod']  
Formula: y ~ A + (1 | Block)  
Data: data.rcb1  
Control: lmerControl(check.nobs.vs.nRE = "ignore")
```

```
REML criterion at convergence: 712.1
```

```
Scaled residuals:
```

Min	1Q	Median	3Q	Max
-1.78748	-0.57868	-0.07108	0.49991	2.33728

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
Block	(Intercept)	132.57	11.514
Residual		20.91	4.572

```
Number of obs: 105, groups: Block, 35
```

```
Fixed effects:
```

	Estimate	Std. Error	t value
(Intercept)	43.034	2.094	20.55
AB	28.452	1.093	26.03
AC	40.156	1.093	36.74

# What about lmer?

```
> anova(data.rcb1.lmer)
```

Analysis of Variance Table

	Df	Sum Sq	Mean Sq	F value
A	2	29855	14927	714.03

# What about lmer?

```
> # Perform SAS-like p-value calculations (requires the lmerTest and pbkrtest)
> library(lmerTest)
> data.rcb1.lmer <- update(data.rcb1.lmer)
> summary(data.rcb1.lmer)
```

Linear mixed model fit by REML t-tests use Satterthwaite approximations to degrees of freedom [using Eigen and Eigenpack]  
lmerMod]

Formula: y ~ A + (1 | Block)

Data: data.rcb1

Control: lmerControl(check.nobs.vs.nRE = "ignore")

REML criterion at convergence: 712.1

Scaled residuals:

Min	1Q	Median	3Q	Max
-1.78748	-0.57868	-0.07108	0.49991	2.33728

Random effects:

Groups	Name	Variance	Std.Dev.
Block	(Intercept)	132.57	11.514
Residual		20.91	4.572

Number of obs: 105, groups: Block, 35

Fixed effects:

Estimate	Std. Error	df	t value	Pr(> t )
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# What about lmer?

```
> anova(data.rcb1.lmer) # Satterthwaite denominator df method
```

Analysis of Variance Table of type III with Satterthwaite approximation for degrees of freedom

	Sum Sq	Mean Sq	NumDF	DenDF	F.value	Pr(>F)
A	29855	14927	2	68	714.03	< 2.2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```
> anova(data.rcb1.lmer,ddf="Kenward-Roger") # Kenward-Roger denominator df
```

Analysis of Variance Table

	Df	Sum Sq	Mean Sq	F value
A	2	29855	14927	714.03

# Section 2

## Worked Examples

# Worked Examples