

# Workshop 9.3a: Randomized block designs

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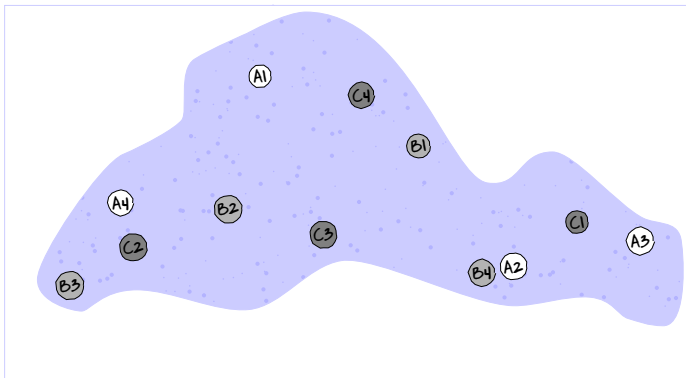
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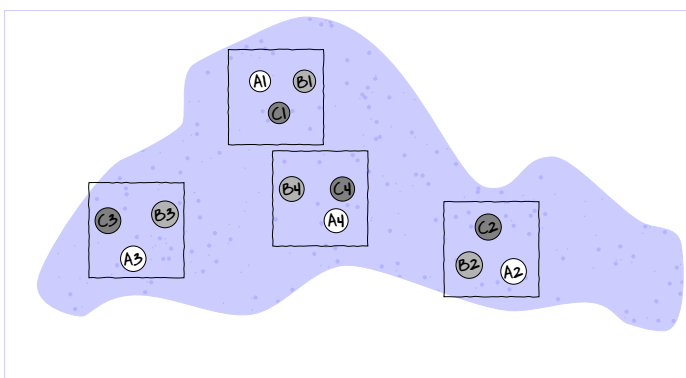
## 1. Randomized Block (RCB) designs

### 1.1. RCB design

Simple



Randomized block design



### 1.2. RCB design

$$y_{ij} = \mu + \beta_i + \alpha_j + \varepsilon_{ij}$$

$\mu$  - the mean of the first treatment group

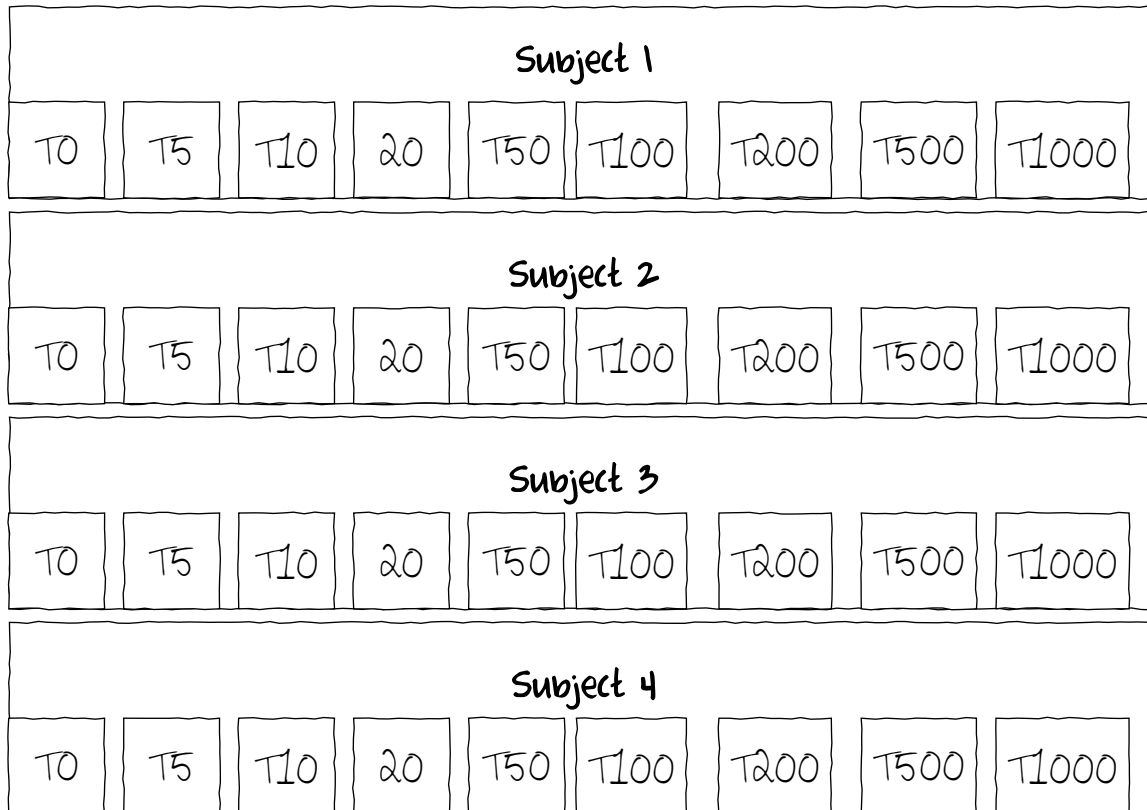
$\beta$  - the random (Block) effect

$\alpha$  - the main within Block effect

e.g.

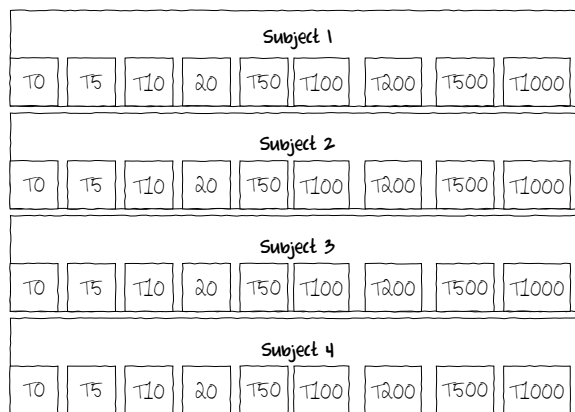
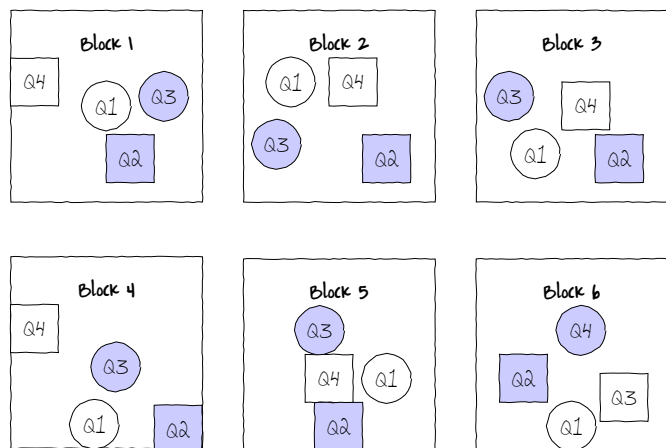
*abundance = base + block + treatment*

### 1.3. Repeated measures designs



### 1.4. Assumptions

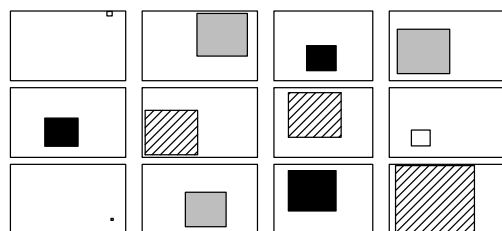
- Normality, homogeneity of variance
- No Block by within-block interaction



### 1.5. Assumptions

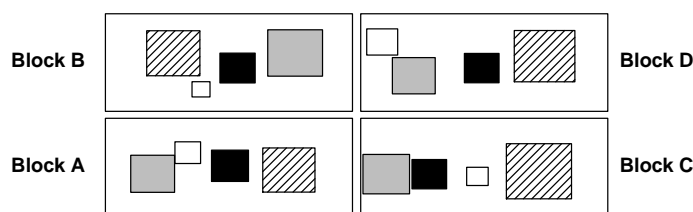
- Normality, homogeneity of variance
- No Block by within-block interaction
- **Independence**
  - (variance-covariance structure)

### 1.6. Var-cov structure



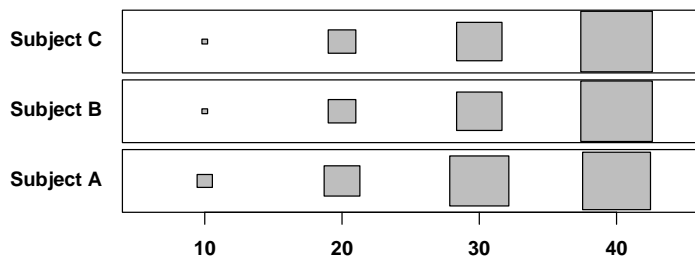
□ T1 ■ T2 ◐ T3 ▨ T4

	T1	T2	T3	T4
T1	0.15	0.00	0.00	0.00
T2	0.00	0.15	0.00	0.00
T3	0.00	0.00	0.15	0.00
T4	0.00	0.00	0.00	0.15



□ T1 ■ T2 ◐ T3 ▨ T4

	T1	T2	T3	T4
T1	0.15	0.05	0.05	0.05
T2	0.05	0.15	0.05	0.05
T3	0.05	0.05	0.15	0.05
T4	0.05	0.05	0.05	0.15



Time (mins)

	T1	T2	T3	T4
T1	0.15	0.60	0.30	0.10
T2	0.60	0.15	0.60	0.30
T3	0.30	0.60	0.15	0.60
T4	0.10	0.30	0.60	0.15

## 1.7. Assumptions

- Normality, homogeneity of variance
- No Block by within-block interaction
- Independence
  - (variance-covariance structure)
    - \* **RCB** - usually met
    - \* **Repeated measures** - rarely met

## 1.8. Example

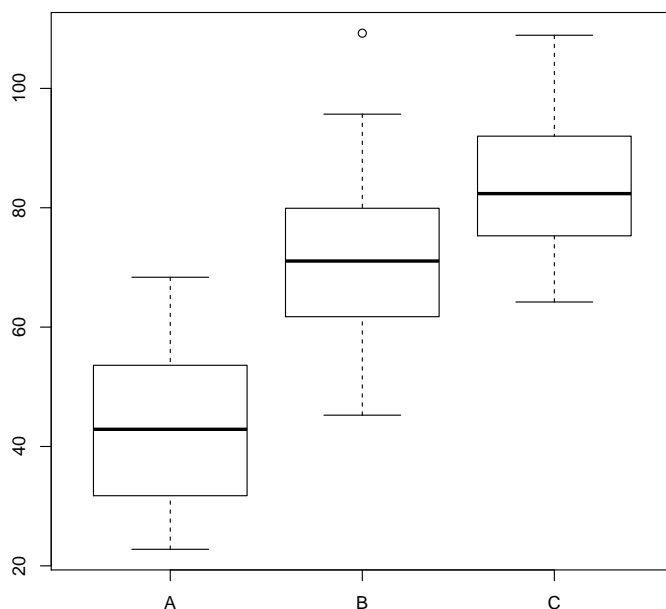
```
> data.rcb1 <- read.csv('../data/data.rcb1.csv', strip.white=TRUE)
> head(data.rcb1)
```

```
      y A Block
1 37.39761 A   B1
2 61.47033 B   B1
3 78.07370 C   B1
4 30.59803 A   B2
5 59.00035 B   B2
6 76.72575 C   B2
```

## 1.9. Exploratory data analysis

Normality and homogeneity of variance

```
> boxplot(y~A, data.rcb1)
```



## 1.10. Exploratory data analysis

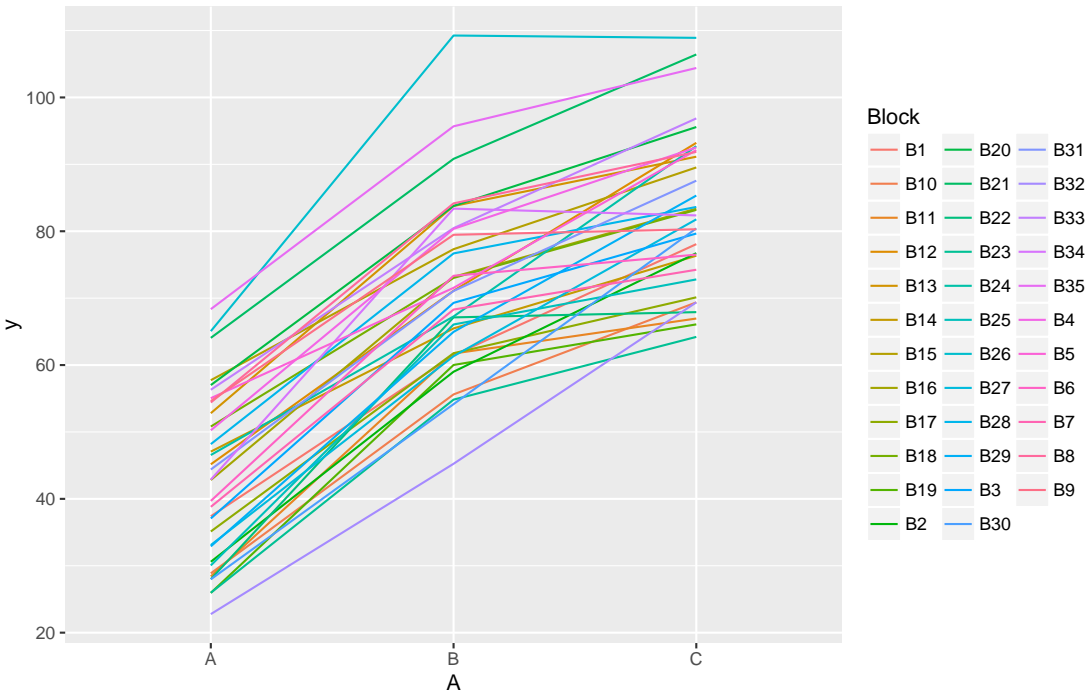
No block by within-block interaction



```

> library(ggplot2)
> ggplot(data.rcb1, aes(y=y, x=A, group=Block,color=Block)) +
+   geom_line() +
+   guides(color=guide_legend(ncol=3))

```



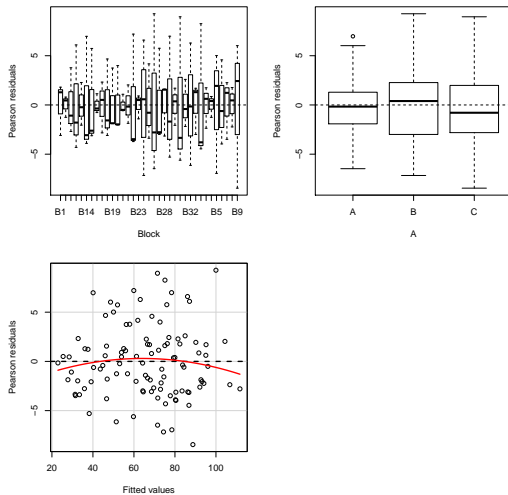
### 1.11. Exploratory data analysis

No block by within-block interaction

```

> library(car)
> residualPlots(lm(y~Block+A, data.rcb1))

```

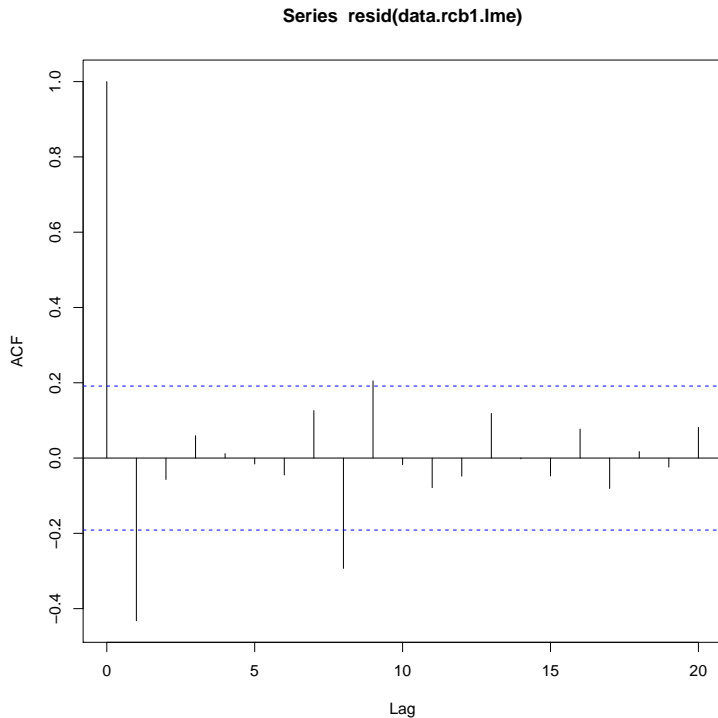


	Test stat	Pr(> t )
Block	NA	NA
A	NA	NA
Tukey test	-0.885	0.376



### 1.12. Sphericity

```
> library(nlme)
> data.rcb1.lme <- lme(y~A, random=~1|Block,
+ data=data.rcb1)
> acf(resid(data.rcb1.lme))
```



### 1.13. Model fitting

```
> #Assuming sphericity
> data.rcb1.lme <- lme(y~A, random=~1|Block, data=data.rcb1,
+ method='REML')
> data.rcb1.lme1 <- lme(y~A, random=~A|Block, data=data.rcb1,
+ method='REML')
> AIC(data.rcb1.lme, data.rcb1.lme1)
```

	df	AIC
data.rcb1.lme	5	722.1087
data.rcb1.lme1	10	727.2001

```
> anova(data.rcb1.lme, data.rcb1.lme1)
```

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
data.rcb1.lme	1	5	722.1087	735.2336	-356.0544			
data.rcb1.lme1	2	10	727.2001	753.4499	-353.6001	1 vs 2	4.908574	0.4271

### 1.14. Model fitting

```
> #Assuming sphericity
> data.rcb1.lme.AR1 <- lme(y~A, random=~1|Block, data=data.rcb1,
+ correlation=corAR1(),method='REML')
> data.rcb1.lme1.AR1 <- lme(y~A, random=~A|Block, data=data.rcb1,
+ correlation=corAR1(),method='REML')
> AIC(data.rcb1.lme, data.rcb1.lme1,data.rcb1.lme.AR1, data.rcb1.lme1.AR1)
```



```

df      AIC
data.rcb1.lme      5 722.1087
data.rcb1.lme1    10 727.2001
data.rcb1.lme.AR1  6 723.3178
data.rcb1.lme1.AR1 11 729.2001

```

```
> anova(data.rcb1.lme, data.rcb1.lme1,data.rcb1.lme.AR1, data.rcb1.lme1.AR1)
```

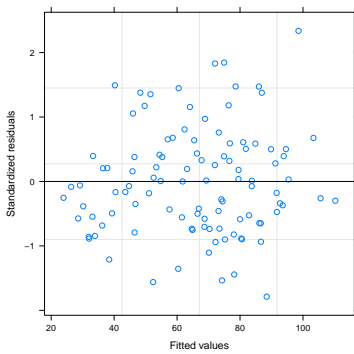
```

Model df      AIC      BIC    logLik  Test  L.Ratio p-value
data.rcb1.lme      1  5 722.1087 735.2336 -356.0544
data.rcb1.lme1     2 10 727.2001 753.4499 -353.6001 1 vs 2 4.908574 0.4271
data.rcb1.lme.AR1  3  6 723.3178 739.0676 -355.6589 2 vs 3 4.117606 0.3903
data.rcb1.lme1.AR1 4 11 729.2001 758.0748 -353.6001 3 vs 4 4.117606 0.5326

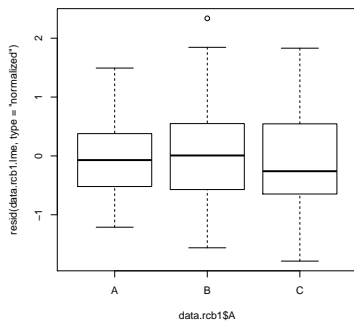
```

### 1.15. Model validation

```
> plot(data.rcb1.lme)
```



```
> plot(resid(data.rcb1.lme, type='normalized') ~
+ data.rcb1$A)
```

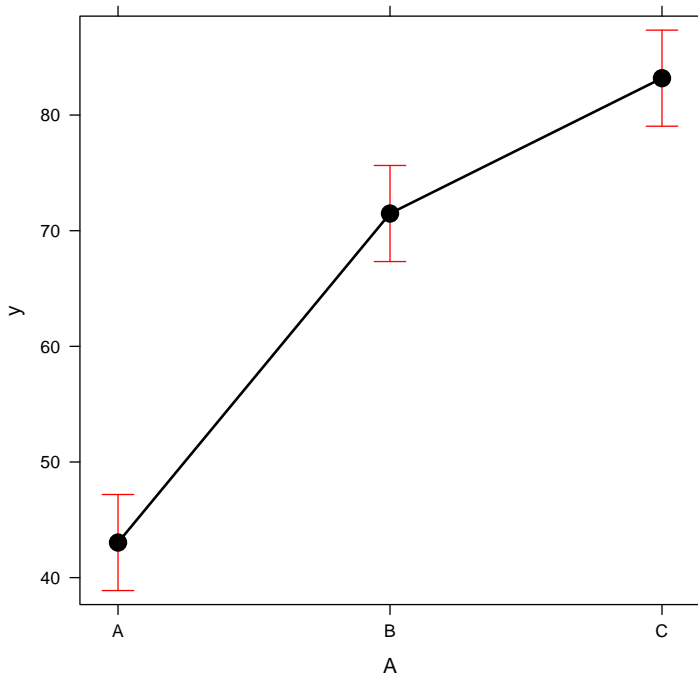


### 1.16. Effects plots

```
> library(effects)
> plot(Effect('A',data.rcb1.lme))
```



A effect plot



### 1.17. Parameter estimates

```
> summary(data.rcb1.lme)
```

Linear mixed-effects model fit by REML

Data: data.rcb1

AIC	BIC	logLik
722.1087	735.2336	-356.0544

Random effects:

Formula: ~1 | Block

(Intercept) Residual

StdDev:	11.51409	4.572284
---------	----------	----------

Fixed effects: y ~ A

	Value	Std.Error	DF	t-value	p-value
(Intercept)	43.03434	2.094074	68	20.55053	0
AB	28.45241	1.092985	68	26.03185	0
AC	40.15556	1.092985	68	36.73936	0

Correlation:

(Intr) AB

AB -0.261

AC -0.261 0.500

Standardized Within-Group Residuals:

Min	Q1	Med	Q3	Max
-1.78748258	-0.57867597	-0.07108159	0.49990644	2.33727672

Number of Observations: 105

Number of Groups: 35





### 1.18. Parameter estimates

```
> intervals(data.rcb1.lme)
```

Approximate 95% confidence intervals

```
Fixed effects:
      lower      est.      upper
(Intercept) 38.85568 43.03434 47.21300
AB          26.27140 28.45241 30.63343
AC          37.97455 40.15556 42.33658
attr("label")
[1] "Fixed effects:"
```

```
Random Effects:
Level: Block
      lower      est.      upper
sd((Intercept)) 8.964236 11.51409 14.78925
```

```
Within-group standard error:
      lower      est.      upper
3.864944 4.572284 5.409077
```

### 1.19. Parameter estimates

```
> VarCorr(data.rcb1.lme)
```

```
Block = pdLogChol(1)
      Variance StdDev
(Intercept) 132.57434 11.514093
Residual    20.90578  4.572284
```

### 1.20. ANOVA table

```
> anova(data.rcb1.lme)
```

	numDF	denDF	F-value	p-value
(Intercept)	1	68	1089.3799	<.0001
A	2	68	714.0295	<.0001

### 1.21. R<sup>2</sup>

[1] 0.6516126

[1] 0.300933

[1] 0.04745443

[1] 0.9525456



### 1.22. What about lmer?

```
> library(lme4)
> data.rcb1.lmer <- lmer(y~A+(1|Block), data=data.rcb1, REML=TRUE,
+   control=lmerControl(check.nobs.vs.nRE="ignore"))
> data.rcb1.lmer1 <- lmer(y~A+(A|Block), data=data.rcb1, REML=TRUE,
+   control=lmerControl(check.nobs.vs.nRE="ignore"))
> AIC(data.rcb1.lmer, data.rcb1.lmer1)
```

```
df      AIC
data.rcb1.lmer    5 722.1087
data.rcb1.lmer1  10 727.2001
```

```
> anova(data.rcb1.lmer, data.rcb1.lmer1)
```

```
Data: data.rcb1
Models:
data.rcb1.lmer: y ~ A + (1 | Block)
data.rcb1.lmer1: y ~ A + (A | Block)
      Df      AIC      BIC logLik deviance Chisq Chi Df Pr(>Chisq)
data.rcb1.lmer    5 729.03 742.30 -359.51  719.03
data.rcb1.lmer1  10 733.98 760.52 -356.99  713.98 5.0529    5    0.4095
```

### 1.23. What about lmer?

```
> summary(data.rcb1.lmer)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: y ~ A + (1 | Block)
Data: data.rcb1
Control: lmerControl(check.nobs.vs.nRE = "ignore")
```

REML criterion at convergence: 712.1

```
Scaled residuals:
      Min       1Q   Median       3Q      Max
-1.78748 -0.57868 -0.07108  0.49991  2.33728
```

```
Random effects:
Groups   Name      Variance Std.Dev.
Block   (Intercept) 132.57   11.514
Residual                20.91    4.572
Number of obs: 105, groups: Block, 35
```

```
Fixed effects:
      Estimate Std. Error t value
(Intercept)  43.034     2.094   20.55
AB           28.452     1.093   26.03
AC           40.156     1.093   36.74
```

```
Correlation of Fixed Effects:
(Intr) AB
AB -0.261
AC -0.261  0.500
```



### 1.24. What about lmer?

```
> anova(data.rcb1.lmer)
```

Analysis of Variance Table

	Df	Sum Sq	Mean Sq	F value
A	2	29855	14927	714.03

### 1.25. What about lmer?

```
> # Perform SAS-like p-value calculations (requires the lmerTest and pbkrtest package)
> library(lmerTest)
> data.rcb1.lmer <- update(data.rcb1.lmer)
> summary(data.rcb1.lmer)
```

Linear mixed model fit by REML t-tests use Satterthwaite approximations to degrees of freedom [lmerMod]

Formula: y ~ A + (1 | Block)

Data: data.rcb1

Control: lmerControl(check.nobs.vs.nRE = "ignore")

REML criterion at convergence: 712.1

Scaled residuals:

	Min	1Q	Median	3Q	Max
	-1.78748	-0.57868	-0.07108	0.49991	2.33728

Random effects:

Groups	Name	Variance	Std.Dev.
Block	(Intercept)	132.57	11.514
Residual		20.91	4.572

Number of obs: 105, groups: Block, 35

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	43.034	2.094	40.930	20.55	<2e-16 ***
AB	28.452	1.093	68.000	26.03	<2e-16 ***
AC	40.156	1.093	68.000	36.74	<2e-16 ***

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr)	AB
AB	-0.261	
AC	-0.261	0.500

### 1.26. What about lmer?

```
> anova(data.rcb1.lmer) # Satterthwaite denominator df method
```

Analysis of Variance Table of type III with Satterthwaite approximation for degrees of freedom

	Sum Sq	Mean Sq	NumDF	DenDF	F.value	Pr(>F)
A	29855	14927	2	68	714.03	< 2.2e-16 ***

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1



```
> anova(data.rcb1.lmer,ddf="Kenward-Roger") # Kenward-Roger denominator df method
```

Analysis of Variance Table

	Df	Sum Sq	Mean Sq	F value
A	2	29855	14927	714.03

## 2. Worked Examples

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### 2.0. Worked Examples